2.

Phase Measurement By Superposition

2.1 Introduction

The method of vector diagrams for determining the magnitudes and relative phases of voltages and currents in a-c networks is not easy in many cases. Consider, for example, the following circuit.

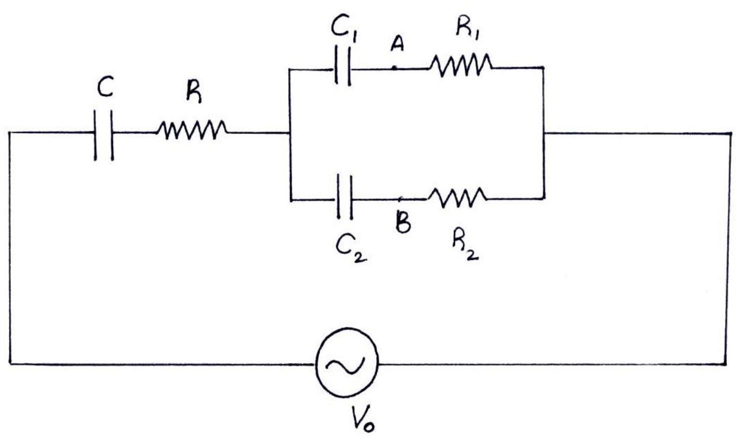

Figure 2.1:

If you want to know the phase difference between the voltages V_A and V_B

it would not at all be a simple matter to infer this from a vector diagram.(Try it). For this reason, we introduce a method by which you can determine phase differences *directly* and study phase relationships in various networks.

2.2 Principle of measurement

The principal we shall employ is *superposition* of the voltage to be measured and a *fixed standard voltage* so that the phase is determined relative to that of the standard voltage. We shall refer to this superposition as 'mixing'.

The fixed(standard) voltage may be derived from any coherent source*; in this board it is the output of a transformer. Suppose we wish to measure the phase of a signal V_i different, say higher, from the voltage V_s of the standard signal; this is connected to a potential divider** P(with a variable pot) and the output adjusted to give exactly the same magnitude as the standard voltage.

We can now mix the two voltages so that we get either $V + V_S$ or $V - V_S$. This can be easily done by first mixing the voltages in one way as in figure 2.2(b) and then reversing the polarity of one of the two voltages for mixing figure 2.2(c). On vector diagrams the two cases are illustrated below:

We can measure the magnitude of the resultant in the two cases. From figure 2.3 it can be seen(derive these results yourself) that

$$V^- = 2V \sin\frac{\delta}{2} \tag{2.1}$$

and

$$V^+ = 2V \cos\frac{\delta}{2} \tag{2.2}$$

so that

$$\frac{V^-}{V^+} = \tan\frac{\delta}{2} \tag{2.3}$$

This immediately gives the phase difference δ. In this way, all phases are determined relative to the coherent standard.

*i.e, the standard source and the voltage to be measured have a phase difference that does not change with time. See Appendix I for this.

**The resistance of the potential divider should be sufficiently high(it is about 100k in the network designed by us) so that it does not disturb the phase or magnitude of the voltage measured

There may be cases when the voltage to be measured is smaller than the standard voltage. In this case, *we drop the standard voltage* across the potential divider until it has the same magnitude as the voltage to be measured and then the mixing is carried out.

2.3 Network Board

A schematic diagram of the lay out of the board is given below:

The board provides the coherent standard voltage from a transformer output(about 20volts). There is a switch for reversing its polarity, i.e. for changing its phase by π.

There is a set of four resistors of equal value(20k) in series connected to a variable pot(20k) so that a continuous adjustment of voltage is possible from the voltage divider.

You are also provided a whole range of L, C and R values and a voltmeter. There is another transformer which provides different voltages for applying to any network you may construct and study.

2.4 Experiment A

To study the relative phases of voltages across resistors and capacitors in series

A supply of about 100volts is connected to as series of resistors. Choose a voltage across one of the resistors (conveniently about 30volts) and measure its phase in the manner suggested; i.e. by dropping to the 'standard voltage' and mixing it. Repeat this for the other resistors. What information do you get about the phase relationships amongst the voltages across the various resistors?

Now do the same for a number of capacitors in series. Interpret your results.

2.5 Experiment B

To measure the phase difference between V_R and V_C in a simple RC circuit. Connect a resistor and a capacitor in series to a source V_o.

Measure the phase difference between V_R and V_S (i.e between V_R and V_o. If the outputs of the two transformers are in phase, the phase of V_R relative to V_S and V_o would be identical). Similarly, determine the phase difference between V_C and V_S. See if the phase difference between V_R and V_C inferred from this is as should be. Here some confusion may arise in the calculation of δ_R,δ_C and δ_{RC} (shown in figure 2.5). What you actually measure are the V^+ and V^- values in each case. Now you may find yourself in a dilemma, whether to divide V^+ by V^- or V^- by V^+ to calculate δ_R (or δ_C) and whether δ_R and δ_C are to be added or subtracted(from each other) to obtain $\delta_R C$. The following vector diagram will help you to sort out this.

From the figure it can be easily seen that

$$\tan\frac{\delta_R}{2} = \frac{V_1^-}{V_1^+}; \tan\frac{\delta_C}{2} = \frac{V_2^-}{V_2^+} \tag{2.4}$$

and

$$\delta_{RC} = \delta_R + \delta_C \tag{2.5}$$

Thus, in both cases divide V^- (the resultant of smaller magnitude) by V^+(the resultant of larger magnitude) to obtain δ_R and δ_C. Now add the two to get δ_{RC}. Similar vector diagrams can help resolving this tangle in any other case as well. Try this for a variety of values for R and C.

The phase difference between V_o and V_R is given by $\tan^{-1}\omega CR$.
Verify this.

2.6 Experiment C

To study more about phase relationships in an RC network

With a capacitor and a set of resistors in series measure the phases(relative to the standard) of V_C, V_{CR_1},V_{CR_2} etc.

Represent them by vectors starting from the same point. See also if you can construct a polygon with the vectors. You can take a number of capacitors in series with a resistor and repeat this experiment. Now you can make measurements on capacitors and resistors connected in different ways. For instance, measure the phase difference V_{in} and V_{out} in the circuit given below.

Interpret your results.

The RC combination is sometimes used for shifting the phase of a signal. In the RC network [Figure 2.7] it can be shown that a phase shift of 180° occurs for frequency $f = \frac{1}{2\pi RC\sqrt{6}}$. *For 50hz,the value of RC turns out to be about 1.3msec. Choose 1μf capacitors and a value of R around 1.3k and make up such a circuit. Now measure directly the phase difference between* V_{in} *and* V_{out}. *In addition to the phase shift there is also attenuation of the signal so that if* V_{in} *is 220volts,* V_{out} *will turn out to be only 7volts.*

You may also measure the phase differences between V_{in} *and the voltages after the first pair CR(i.e. across the first resistor) and after the second pair CR. See if the phase changes by 60° each time. (It should not!)*

If you measure the phase difference between V_C *and* V_R*, say the first pair following the input, you will find that it is not 90° as you may imagine at first glance. Can you explain this?*

2.7 Experiment D

To study the phase relationships in an LR circuit

Connect an inductor and a resistor in series to a source V_o. Measure the phase difference between V_o and V_L, V_o and V_R. You can determine, from this, the phase difference δ between V_L and V_R. This will not be $\pi/2$ since there is power loss in the inductor. You can easily show(try this) that δ is given by

$$\tan\left(\frac{\pi}{2} - \delta\right) = \frac{r}{\omega L} \tag{2.6}$$

where r is the effective power loss resistance of L. Compare the value r you obtain this way to the value obtained by triangulation. An agreement within a factor of two is to be considered satisfactory.

The phase difference between V_o and V_R is given by $\tan^{-1}(\frac{\omega L}{R+r})$
Verify this for a large number of combinations.

2.8 Experiment E

To study the phase of V_R in an LCR circuit

Connect an LCR circuit in series. Measure the phase difference between

V_R and V_o. This phase difference is given by $\tan^{-1}(\frac{\omega^2 LC-1}{\omega C(R+r)}$ and will be zero at resonance. Verify this by varying C and then study how δ_{OR} changes as you pass through resonance, i.e. as you go from LC $< \frac{1}{\omega^2}$ to LC $> \frac{1}{\omega^2}$

2.9 Experiment F

To study the phase relationships amongst various voltages in an LCR circuit

In an LCR circuit, measure the phase differences $\delta(V_o, V_{Lr})$ and $\delta(V_o, V_C)$ and hence determine $\delta(V_C, V_{Lr})$. Measure these phase differences directly for various values of C and calculate $\delta(V_C, V_{Lr})$ in each case. It ought to be independent of C. You may also infer the phase difference between V_L and V_o. This will be given by $\tan^{-1}(\frac{R+r}{\omega L})$.

The phase of V_{Lr} relative to that V_o is given by

$$\tan^{-1}(\frac{R/\omega L}{1+\frac{r(R+r)}{\omega^2 L^2}}) \tag{2.7}$$

(Establish the relation). Check this from your results.

2.10 Experiment G

To design equivalent circuits for 'hybrid' RC networks.

Construct a network as follows:

Measure the phases of the currents in different branches of the circuit. This is done by measuring the phases of V_{R_1}, V_{R_2} and V_r where r is a small resistance that you may add in the C branch to measure the phase of the current in that branch.

From the data you have, reconstruct an equivalent RC series circuit. Wire up an actual circuit with these components, measure the magnitude of Z(the impedance), the current and its phase. Verify the equivalence.

You may try other circuit combinations yourself and establish equivalent circuits.

2.11 Experiment H

To measure the phase of the voltage across any two points in a complex network

Measure the phase difference between V_A and V_B in the network shown in figure 2.1. You may try other networks where it is very difficult to measure the phase of the voltage across any two points in the network by the method of vector diagrams. In all such cases you will be able to measure it by this method directly.

2.12 Experiment I

To study the phase relationships amongst various voltages in an LCR circuit

In an LCR circuit measure the phase differences $\delta(V_o, V_{Lr})$ and $\delta(V_o, V_C)$. Hence determine $\delta(V_C, V_{Lr})$. Do this for various values of C. Check if $\delta(V_C, V_{Lr})$ is independent of C. You may infer also the phase difference $\delta(V_o, V_L)$ though its not directly measurable(because of r).

Figure 2.9 gives the full line triangle for voltages V_o, V_R and V_{LCr}, while the phase angles measured are $\delta(V_o, V_{Lr}) = \chi$ and $\delta(V_o, V_C) = \theta$. The deduced angle is $\delta(V_C, V_{Lr}) = \phi = \theta - \chi$. From figure 2.9 we note that

$$\tan\delta(V_o, V_C) = \tan\theta = \frac{I_o(R+r)}{I_o(\omega L - \frac{1}{\omega C})} = \frac{R+r}{\omega L - \frac{1}{\omega C}} \tag{2.8}$$

and

$$\tan\phi = \frac{I_o r}{I_o \omega L} = \frac{r}{\omega L} \tag{2.9}$$

From these expressions one can deduce the expression for $\delta(V_C, V_{Lr}) = \chi$, since $\chi = \theta - \phi$.

APPENDIX-I

Need For Coherent Sources

In this board we have used a *coherent* standard voltage for mixing with the voltage whose phase is to be determined. Now the phase difference between these two voltages ought to stay constant over the time you take to make these measurements. In general, phases of line voltages hardly maintain constancy over such long periods of time and it is of no use to compare phases of two a-c voltages which are entirely independent of each other. It is for this reason that in the board, the standard voltage, namely the output of a transformer, is deprived from the line which also feeds the network of the board so that in spite of line fluctuations, *phase differences* in your experiment remains constant.

This idea, namely that coherent sources have to be derived from the same origin, is akin to the one you come across in interference experiments in physical optics such as, for example, Fresnel's biprism. In the case of light, phase changes of a source occur in about 10^{-8} sec. Since the frequency of visible light is about $6x10^{11}$hz, this means phase fluctuations occur in some 10^{6} cycles. Despite this, the stability is poor since our eye is unable to follow variations in such a short time which is the reason why you do not observe interference patterns with independent sources.

The a-c line supply normally achieves a stability of about 1% in frequency. Thus if you spend 10minutes in taking your readings, your observations last some 30000 cycles and the uncertainty in phase is many times a full cycle. needless to say, two such sources can hardly be coherent over period of measurement.

If you can manage it, try to get two separate audio oscillators tuned to the same frequency and convince yourself that they are not coherent.

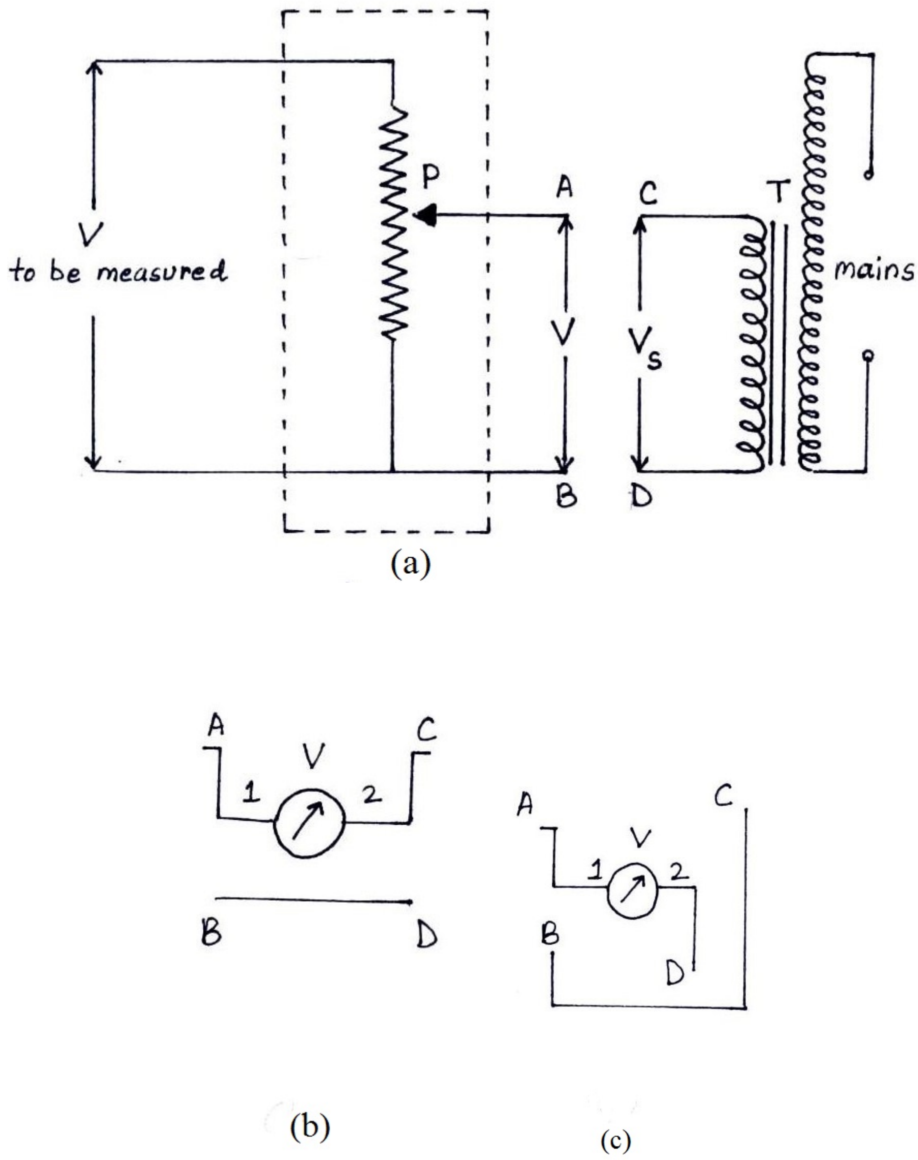

Figure 2.2: (a) An arrangement to achieve $|V| \equiv |V_S|$ and mix the two voltages. (b) and (c) Voltage measurements after superposition and reversal of polarity of one of the voltages

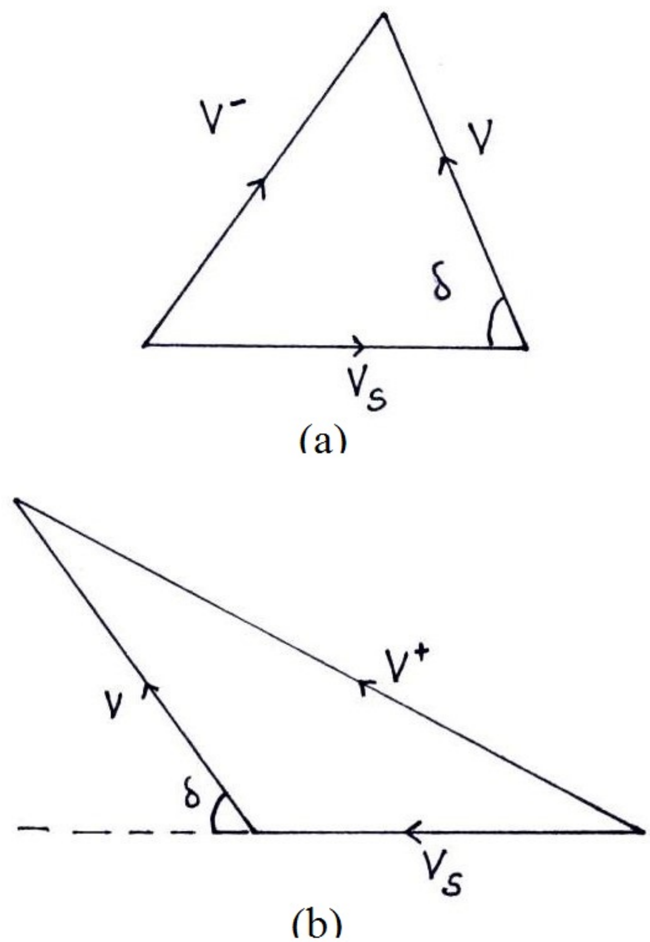

Figure 2.3:

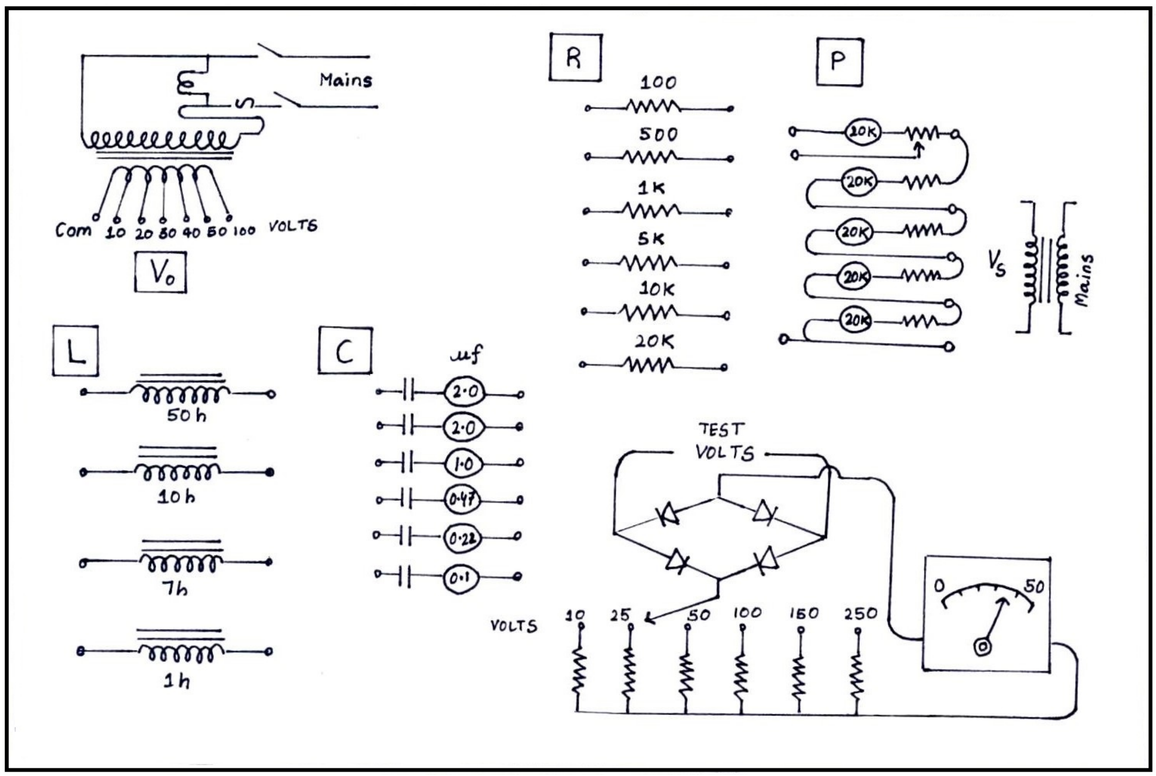

Figure 2.4:

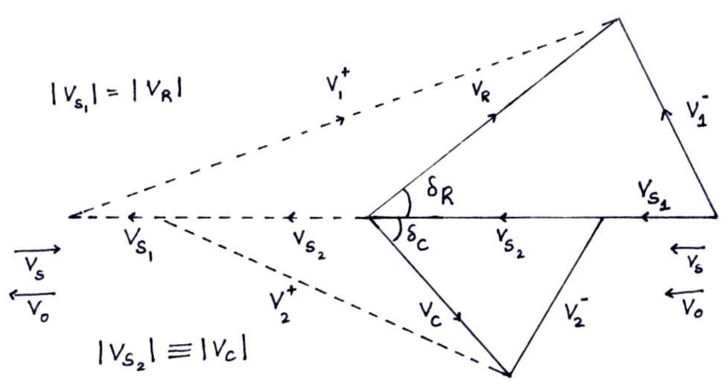

Figure 2.5: You can see that (a)Phase of V_R (or V_C) is same relative to V_s and V_C (b) δ_{RC}=$\delta_R + \delta_C$

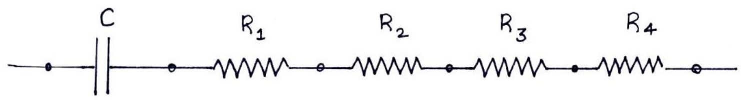

Figure 2.6:

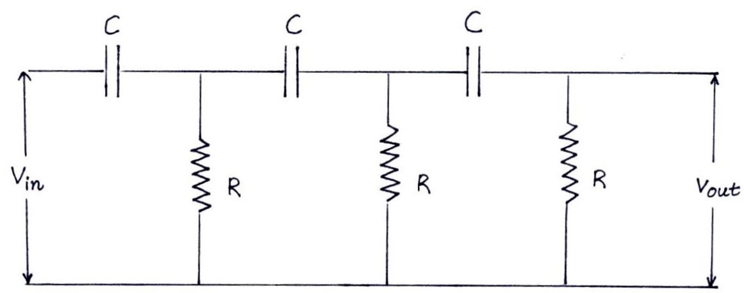

Figure 2.7:

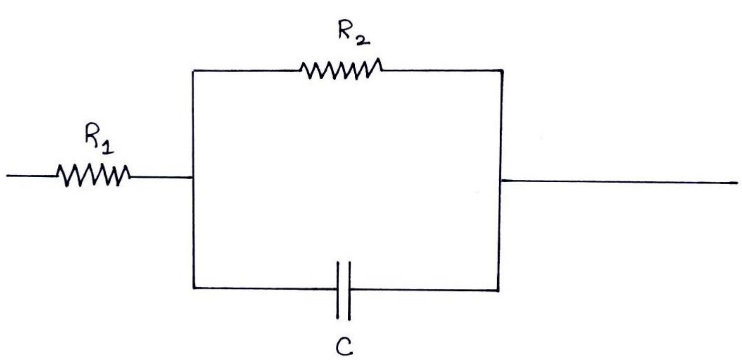

Figure 2.8:

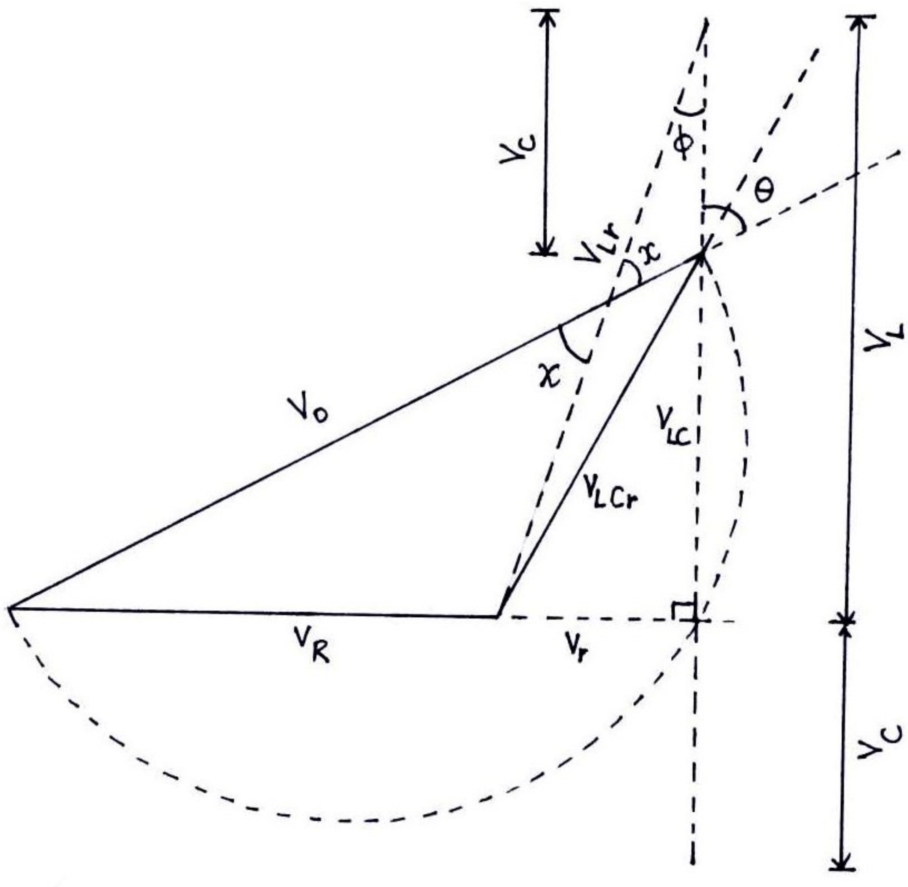

Figure 2.9: