5. Charging and discharging of a capacitor

5.1 Capacitors

A system of charges, physically separated, has potential energy. The simplest example is that of two metal plates of large area carrying opposite
charges so that the potential difference is V. The energy stores is $\frac{1}{2}CV^2$ where C is the capacitance of the system. It is defined as the charge(on either plate) per unit potential difference and depends essentially on the geometry of the system. In the above case the capacitance is given by

$$C = \epsilon_o \frac{A}{d}$$

in mks units, where A is the area(in meter$^2$), d is the separation(in meters), $\epsilon_o$ is a constant (8.85 X 10$^{-12}$ in MKS units) and the unit of capacitance is a farad.( Refer to any standard text for the derivation of this formula).

A system, such as the above one, is called a condenser or, in modern parlance, simply a capacitor. We shall adopt the modern usage. It must not be assumed that a capacitor is always a set of plane parallel plates. Many other geometrical arrangements may be used and often are more practical(See Appendix I).

### 5.2 RC Circuit

The energy may be delivered by a source to a capacitor or the stored energy in a capacitor may be released in an electrical network and delivered to a load. For example, look at the circuit in Figure 5.2. If you turn the switch

![Figure 5.2:](image)

$S_1$ on, the capacitor gets charged and when you turn on the switch $S_2(S_1$
off) the capacitor gets discharged through the load. The rate at which the charge moves, i.e. the current; this, of course, will depend on the resistance offered. It will be seen, therefore, that the rate of energy transfer will depend on RC where C is the capacitance and R some effective resistance in the circuit. It can be shown (Appendix II) that the charging of a capacitor can be represented by the relation

\[ q = q_0(1 - e^{-t/RC}) \]  

(5.2)

where \( q \) is the charge on the plates at time \( t \); similarly, the discharge occurs according to the relation

\[ q = q_0e^{-t/RC} \]  

(5.3)

Thus, the rate at which the charge or discharge occurs depends on the 'RC' of the circuit. The exponential nature of the charging and discharging processes of a capacitor is obvious from equation 5.2 and 5.3. You would have ample opportunity to learn more about it through the experiments that follow. From equation 5.3 it can be seen that RC is the time during which the charge on the capacitor drops to 1/e of the initial value. Further, since RC has dimensions of time, it is called the time constant of the circuit.

In the following series of experiments, you will study the time variation of charge, voltage and energy in an RC circuit.

### 5.3 The Network Board

The network board for these experiments consists of a number of resistors and capacitors and two d-c meters. The centrally pivoted meters facilitate measurements during both charging and discharging of a capacitor. Figure 5.3 shows the scheme of arrangement of these on the board and their connections underneath it. The capacitors are of electrolytic type (since you need high values of capacitance). These are meant for use with d-c power and great care must be taken to connect them with the right polarity.

In order to make the time constant RC of the circuit large the resistors also need to have high values and are, therefore, of carbon film type. Remember, the values marked on both R and C are not absolutely dependable. The resistance values are given within ±2% but the capacitance values have a tolerance of ±10% or more.

A regulated d-c power-supply and a stopwatch are also provided along with the board. Use of 20 to 25 volts from this supply would enable you
Figure 5.3:

to reduce the unwanted discharge through the voltmeter and considerably improve the performance of the experiments. This would be obvious from the following discussion. If you look at Figure 5.4 relating to the discharging of a capacitor, you would realize that on turning the switches $S_1$ and $S_2$ on, the capacitor would discharge through both the load $R$ and the voltmeter $V$. If $R_v$ be the resistance of the meter, the effective leakage resistance $R'$ would be given by

$$R' = R \frac{R_v}{R + R_v}$$

The unwanted discharge through the meter can, therefore, be reduced only by making $R_v$ much higher than $R$. This is accomplished in a simple way by using a higher voltage source and employing a higher range of the meter for detection. However, even this would not be adequate in case of smaller $C$ values where you should employ a sort of 'sampling method' for voltage measurements. This consists in turning on the switch $S_2$ only at the instant
when a measurement is to be made. You may find it difficult to read the meter, say every 2 seconds or so. In that case, take one set of readings at 0.6, 12, 18...sec., then the next set of readings at 2, 8, 14, 20,...sec. and so on until you have a complete set of readings every 2 seconds.

5.4 Experiment A

To study the charging of a capacitor in an RC circuit

Take a resistor and a capacitor and complete the circuit as shown. Switch on the stop watch and the circuit simultaneously. Read the voltmeter every 2 second until the voltmeter indicates a maximum value $V_0^*$. You may find it difficult to read the meter, say every 2 seconds or so. In that case, take one set of readings at 0.6, 12, 18...sec., then the next set of readings at 2, 8, 14, 20,...sec. and so on until you have a complete set of readings every 2 seconds. Plot the voltage $V_c$ across the capacitor as a function of time. Figure 5.6. To analyse the results, proceed as follows. The voltage across a charging

*Theoretically speaking, in the case of a pure capacitor, the voltage across it should become equal to the source voltage $V_0$ when the capacitor is fully charged. In practise, it is very seldom so. This is because there is always a leakage charge across the capacitor
capacitor is given by (see Appendix II).

\[ V = V_o(1 - e^{-t/RC}) \]  

(5.5)

where \( V_o \) is the maximum voltage. Eq 5.5 means that the capacitor charges exponentially. Let us verify these facts. Rewriting Eq 5.5, we get

\[ \frac{V_o - V}{V_o} = e^{-t/RC} \]  

(5.6)

If we now define a time \( T_{1/2} \) at which the voltage is half the maximum i.e. \( V = V_o/2 \), the above expression would reduce to

\[ T_{1/2} = RC \log_2 2 = 2.30RC \]  

(5.7)

This clearly shows that for a given RC the time \( T_{1/2} \) should be constant.

Choosing values for \( (V_o - V)/V_o \) in geometric progression in steps of \( \frac{1}{2} \), the time intervals \( \Delta T_{1/2} \) can be easily shown to be equal. See Figure 5.6

Eq 5.7 could be examined in yet another way. Make some measurements of \( T_{1/2} \) for different RC combinations and plot these versus RC. In theory this should be a straight line; but the rated values of the components (particularly C may be as much as 10% off). Thus, the values as determined by you are probably more reliable than the specified ones.

Alternatively, you may plot \( \log(V_o - V) \) against \( t \) to verify the Eq. 5.5 and the exponential nature of charging of the capacitor. You ought to get a straight line whose slope would give you the value of \(-1/RC\).
5.5 Experiment B

To study the discharging of a capacitor

As shown in Appendix II, the voltage across the capacitor during discharge can be represented by

\[ V = V_o e^{-t/RC} \] (5.8)

You may study this case exactly in the same way as the charging in Expt A. However, remember that for the case of discharge \((V_o - V)/V_o\) has to be replaced by \(V/V_o\) and \(\log(V_o - V)\) by \(\log V\). (why?) You would find that for the same set of \(R\) and \(C\) the time \(T_{1/2}\) and hence the interval \(\Delta T_{1/2}\) have the same value as in Expt A.

In the circuit shown figure 5.7, if the switch is turned on at time \(t=0\) and turned off at \(t = t_1\), the voltage across the input terminals \(AB\) ideally behaves as in figure 5.8. Plot the output across \(PQ\) in the same manner. Once
again, you should train yourself to think of the RC combination as a 'box' with input terminals AB and output terminals PQ. Suppose the circuit in the above question had been on for some time before the switch was suddenly disconnected. Display both the input and the output (voltages) as a function of time. Assume that the 'box' is now wired as follows figure 5.9 Discuss the

Figure 5.7:

input and output when the switch is turned on and later, turned off.

Exercises pertaining to Expts A and B

1. Change the voltage $V_o$ of the power supply and see if, for a given RC, the time $T_{1/2}$ or the time interval $\Delta T_{1/2}$ remains the same. Do you expect it to change?

2. For a known resistance, the time $T_{1/2}$ determines the capacitance. Use this to determine first $C_1$, then $C_2$ and finally the effective capacitance $C$ with both $C_1$ and $C_2$ in parallel figure 5.10.

   Verify the law $C = C_1 + C_2$ where $C$ is the effective capacitance of the combination in parallel. Try this with various resistors R.

3. Use exactly the same method (by measuring $T_{1/2}$) to verify the law

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots
\]

for a set of capacitors in series with a resistor R [Figure 5.11]. Try with
Charging and discharging of a capacitor

Figure 5.8:

Figure 5.9:
4. Charge a set of capacitors connected in series. (Roughly, about 5 times $T_\frac{1}{2}$ will charge the capacitors to the maximum voltage). Measure the voltage across each and establish the law.

$$C_1V_1 = C_2V_2 = C_3V_3 \ldots \ldots$$

(5.10)

5. Connect a set of capacitors in parallel. Measure the current through each of them** after a fixed interval of time either during the charging or

** You will have to use the terminals provided to the left of the capacitors for connecting the current meter in series with the capacitor individually
during the discharging operation and establish the relation:

\[
\frac{C_1}{i_1} = \frac{C_2}{i_2} = \ldots.. \tag{5.11}
\]

(in verifying such relations as 2-9, 2-10 and 2-11, make sure to measure the capacitance yourself and not just trust the rated values).

6. With an RC time of around 30sec., measure the voltage across R as a function of time while charging and discharging the capacitor. Pay particular attention to the polarity of the voltage across R in each case. It is for this reason that the voltmeter provided is centrally pivoted one. You would also notice that with the passage of time the voltage across the resistor goes on falling until it becomes zero when the capacitor is fully charged or discharged. If you use two voltmeters and measure the voltages across R and C simultaneously you can also verify that at all instants of time

\[
V_R + V_C = V_o \tag{5.12}
\]

This is the verification of kirchoff’s law.

5.6 Experiment C

To study the current flow during charging and discharging of a capacitor

The current flowing through an RC circuit is given by (Appendix II)

\[
I = I_o e^{-t/RC} \tag{5.13}
\]

for the charging circuit and

\[
I = -I_o e^{-t/RC} \tag{5.14}
\]

for the discharging circuit. Thus the current follows the same behaviour as the voltage with time except that its direction is opposite in the two cases.

Connecting the milliammeter in series with the resistor and the capacitor[Figure 5.12, study the behaviour of the current in the two cases [Figure 5.13]
Figure 5.12:

Pay particular attention to the reversing of the current in the circuit. This is why a centrally pivoted current meter is provided.

Also, if you connect the voltmeter across R, in addition to the reversing of polarity in the voltage across R, you would discover that the whole of the voltage appears across it when you commence the charging or the discharging. Also verify if the maximum current $I_o$ at the commencement of the charging and the discharging is given by

$$I_o = \frac{V_o}{R} \quad (5.15)$$

Further, you can see that at all instants of time

$$I = \frac{V_R}{R} \quad (5.16)$$

5.7 Experiment D

To estimate the leakage resistance of a given capacitor

Capacitors, once charged, do not maintain their charges indefinitely even when their terminals are left disconnected. (But, they often maintain it for long times. Do not poke your fingers at these terminals. You are always advised to deliberately discharge the capacitor before leaving your experiment). A capacitor loses its charge by leakage either through the dielectric between or the insulators which holds the capacitor electrodes in place. Thus, strictly speaking, any capacitor may be effectively represented as in figure 5.14 where
$R_C$ representing the leakage resistance of the capacitor $C$, is of the order of a few megaohms.

In the case of an ideal capacitor ($R_C = \infty$) when fully charged, the voltage $V_C$ across it should be equal to $V_0$, figure 5.5 and the final value of the charging current $I$ in the circuit (figure 5.12) should be zero. In practice, as you would discover during the course of these experiments, this is not the case. $V_C$ is always less than $V_0$ and the charging current never drops down to zero. It is easy to understand these facts if you remember the true representation of a capacitor (figure 5.14) Take a resistor $R$ and a capacitor $C$ so that the time constant $RC$ is of the order of 10sec. or more. Connect the in series with a milliammeter [figure 5.15(a)] (note how the capacitor has been represented). Turn on the switch and confine your attention to the current meter to observe how the charging current drops with time. After a time ($5RC$ or more) the capacitor is expected to be fully charged and the current to be zero. On
your meter it may indeed appear as if the current has become zero but if you replace the milliammeter by a microammeter of movement 50µA or less[Figure 5.15(b)] you would find a small steady current flowing persistently no matter how long you wait. Measure this leakage current $I_C$. Assuming the voltage across the capacitor to be the same as $V_o$, the supply voltage (this is not quite correct), calculate the order of the leakage resistance $R_C$ by

$$R_C = \frac{V_o}{I_C} \quad (5.17)$$

You must learn to make approximations like these ($V_C \simeq V_o$) and understand why such approximations do not matter when it is only the order of magnitude of a quantity you are interested in. You should further, be able to appreciate the difficulty in measurement of $V_C$ with a meter of finite resistance and hence the importance of the approximation $V_C = V_o$. However, if you are interested in knowing the leakage resistance more precisely you may calculate it as follows:

$$R_C = \frac{V_o}{I_C} - R \quad (5.18)$$

Do you see how the approximation involved in Eq.5.17 is taken care of in Eq.5.18?

If a capacitor of 50µf and a leakage resistance of 2 megaohms, in how much time will the charged capacitor, left to itself, lose half its charge?

You may now connect the voltmeter across C[figure 5.15 and see how the leakage resistance $R_C$ changes. Try to verify your result by calculation.

Figure 5.14: Representation of an actual capacitor
A capacitor of 100 µf has a leakage resistance of 5 megaohms. A voltmeter of resistance 500 kilohms is connected across it to read the voltage. How much time would it take for the voltage to fall to a value 1/e times the initial value? Calculate first neglecting the leakage resistance and then taking it into account.

5.8 Experiment E

To measure the energy dissipated in charging a capacitor

Some energy is spent by the source in charging a capacitor. A part of it is dissipated in the circuit and the remaining energy is stored up in the capacitor. In this experiment we shall try to measure these energies.

With fixed values of C and R measure the current I as a function of time. The energy dissipated in time \( dt \) is given by \( I^2Rdt \). The total energy dissipated is given by

\[
E = \int I^2Rdt = R \int I^2dt
\]

(5.19)

This integral can be evaluated very easily by graphical method as follows:

From the observed values of I plot \( I^2 \) versus \( t \) [figure 5.16]. The area under the curve gives the value of the integral and R times this area is therefore a measure of the energy dissipated in the circuit.

Does the energy dissipated depend on the value of the resistance? A cursory glance at Eq5.19 would indicate that it should. You can test this as follows:

Plot the \( I^2, t \) curves for different values of the resistance R in the circuit and measure the area in each case. You will discover an amazing result—the energy dissipated thus would turn out to be independent of the charging resistance.

In charging or discharging a capacitor through a resistor an energy equal to \( \frac{1}{2}CV^2 \) is dissipated in the circuit and is independent of the resistance in the circuit. Can you devise an experiment to measure it calorimetrically? Try to work out the values of R and C that you would have to employ in this experiment. Remember, capacitors having a high value of capacitance cannot
withstand voltages higher than 50 to 60 volts and those which can withstand higher voltages have lower values for capacitance.

Suppose the total resistance in the circuit including that of the connecting wires is made zero, in what part of the circuit would the energy $\frac{1}{2}CV^2$ be dissipated now? How will you modify your above calorimetric measurement for this case?

Repeating this for the case of discharging, you will find that again an equal amount of energy is dissipated in the circuit. Since this energy in the case of discharging comes from the capacitor you can draw a simple conclusion from these experiments. Of the total energy drawn from the source in charging a capacitor, half is dissipated in the circuit and half is stored up in the capacitor irrespective of the value of the resistance. In other words, of the total energy spent in charging a capacitor you can recover only half of it.

5.9 Experiment F

To study the dependence of the energy dissipated on C and V

For a fixed voltage $V_o$, the energy dissipated is proportional to the value of C i.e. if $E_1$, $E_2$ etc. are the energies dissipated for capacitors $C_1,C_2$ etc., we shall have

$$\frac{E_1}{C_1} = \frac{E_2}{C_2} = \ldots$$

(5.20)

Measure the energies $E_1,E_2$ etc. graphically(Expt E) and check this.

For a fixed capacitance C, estimate similarly the energy dissipated for different values of the supply voltage $V_1,V_2$, etc. You may vary V from 5 to 20 volts or so. Establish the relation

$$\frac{E_1}{V_1^2} = \frac{E_2}{V_2^2} = \ldots$$

(5.21)

In fact, the energy dissipated is $\frac{1}{2}CV^2$(Appendix II); see, if you can verify this in all the experiments discussed.

The result that the energy dissipated($\frac{1}{2}CV^2$) in an RC circuit is independent of R seems strange. Try and see if you can present an argument to justify this. Discuss this in the limiting cases $R \to 0$ and $R \to \infty$ also.
5.10 Experiment E

To study the adiabatic charging of a capacitor

Is there no way of eliminating or reducing the dissipation of energy $\frac{1}{2}CV^2$ in charging of a capacitor? The answer is yes, there is a way. Instead of charging a capacitor to the maximum voltage $V_0$ in a single step if you charge it to this voltage in small steps the dissipation of energy can be reduced. Theoretically speaking, if the successive steps are infinitesimally small the dissipation can be entirely eliminated. This is called adiabatic charging of a capacitor. You can verify this with the following experiment.

Suppose you want to charge a capacitor $C$ to a voltage $V_0$. If you do that in a single step you know (ExptE) that an energy $\frac{1}{2}CV^2$ would have to be dissipated in the circuit. On the other hand, if you charge the capacitor to a voltage $V_0/2$ to $V_0$ the total energy dissipated would be $\frac{1}{4}CV^2$. (Why?) You can check this experimentally. The trick is to first keep the charging voltage to $V_0/2$, let the capacitor charge for a time much greater than RC of the circuit, disconnect the power supply, increase its voltage to $V_0$, reconnect it and let the capacitor charge to $V_0$. Plot $I^2t$ curves for the two parts and find out the total energy dissipated in the process. Compare this with the area of the curve obtained when the capacitor is charged to $V_0$ in a single step and you would find the former to be roughly one-half the area in the latter case. The charging voltage in the two cases can be represented as shown in figure 5.17. Now think how you can reduce this loss further. Check your answer experimentally.

A capacitor of 1000 $\mu$F is connected in series with a resistor of 2 kilohms. Calculate the energy dissipated in charging it to 20 volts in a single step. How many equal steps will you have to employ to cut down this loss to one-tenth its value? Show these steps graphically (as in figure ??) taking care to mark the appropriate value of $\Delta t$.

Can you now think of the ideal charging method to reduce this loss to zero? Would it be possible to accomplish this in practice?
Commercially available capacitors come in various forms for use in simple networks. A common one is the paper capacitor in which a pair of metal foils sandwich a thin paper. The whole assembly is then rolled into a bundle, dipped in wax and sealed against moisture. There may still be some leakage of charge through the paper particularly if the applied voltage is large. A practical consideration for a capacitor is always the voltage it can withstand without breakdown.

The capacitance of the system is somewhat increased when there is a dielectric (such as paper) between the electrodes. Other dielectrics commonly used are mica, ceramics and sometimes plastic films.

It can be seen that by reducing the distance between the electrodes one can increase the capacitance; but one cannot do this indefinitely. For a given voltage, electrical breakdown (i.e. current through the dielectric) occurs if the distance is too small. For example, if air is the dielectric and the capacitor is to withstand 100 volts, a separation of at least 1/10 mm is required. The capacitance of a parallel plate capacitor is given by

\[ C = \epsilon_0 \frac{A}{d} \]  

One can see from this relation (the reader is advised to do this arithmetic) that no more than about 10 pico-farad per sq.cm (1 pico-farad = 10^{-12} farad) can be achieved.

2. Electrolytic Capacitors

Some metals like aluminium, when placed in a suitable electrolyte and made the positive electrode (i.e. aluminium is the positive electrode) from a thin film (about 10^{-6} cm) of oxide. This film has a very high resistance to a flow of current in one direction (from aluminium towards electrode) and a very low one in the reverse direction. Thus, provided we use the aluminium side as the positive one, we can obtain fairly large capacitance, a micro-farad per 10 cm^2 area with this kind of system when the aluminium and the electrolyte form the two electrodes.

Even smaller film thicknesses can be made so that electrolytic capacitors can achieve as high as 10^{-4} farad for 10 cm^2. It is obvious that we cannot use an electrolytic capacitor with a-c unless we ensure that its polarity would not change.
Other limitations are that they have a larger leakage current than the ordinary capacitors, their life is shorter, their capacitance may change somewhat after a few months (even the values marked on the new ones may vary by as much as 20%) and the working voltages for these are lower.

In all the circuits wherein these capacitors have been used they are represented as in [figure 5.19], the curved line representing the negative can.

In using these electrolytic capacitors, remember to connect them with the right polarity and always below the rated voltage of the capacitor.
Analysis of an RC circuit with a source of constant EMF

When a resistor and a capacitor are connected in series to a source of voltage $V_o$, we have

$$V_c + V_R = V_o \tag{5.23}$$

where $V_c$ and $V_R$ are the voltages across C and R. Writing

$$V_c = \frac{Q}{C} \tag{5.24}$$

and

$$V_R = RI = R\frac{dq}{dt} \tag{5.25}$$

where $q$ is the charge on the capacitor and $I$ the current, we have

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{V_o}{R} \tag{5.26}$$

This equation is readily integrated after multiplying by the integrating factor $e^{t/RC}$,

$$q e^{t/RC} = \frac{V_o}{R} \int e^{t/RC} \, dt \tag{5.27}$$

$$qe^{t/RC} = CV_o e^{t/RC} + A \tag{5.28}$$

where $A$ is a constant.

For charging, we assume the initial condition $q=0$ at $t=0$ which establishes the equation

$$q = q_o (1 - e^{-t/RC}) \tag{5.29}$$

where we have put $q_o = CV_o$

Similarly, for discharging, we set $q = q_o = CV_o$ at $t=0$ to give

$$q = q_o e^{-t/RC} \tag{5.30}$$

The potential across the capacitor ($q/C$) follows exactly the same dependence on time as the charge.

The current is

$$I = \frac{dq}{dt} = \frac{q_o}{RC} e^{-t/RC} \tag{5.31}$$
or

\[ I = I_o e^{-t/RC} \]  \hspace{1cm} (5.32)

for the charging circuit and

\[ I = -I_o e^{-t/RC} \]  \hspace{1cm} (5.33)

for the discharging circuit. Thus the current follows the same behaviour with time except that the sign is reversed in the two cases.

When the source charges the capacitor, it does work. This work is simply

\[ W = \int V_o I dt \]  \hspace{1cm} (5.34)

since the rate of doing is \( V_o I \). Using equation (2.32), we have

\[ W = V_o I \int_0^\infty = V_o I_o e^{-t/RC} \]  \hspace{1cm} (5.35)

since \( V_o = \frac{q_o}{C} \) and \( I_o = \frac{q_o}{RC} \).

This can be written in either of the following forms:

\[ W = CV_o^2 = \frac{q_o^2}{C} \]  \hspace{1cm} (5.36)

An interesting point to note is this: when the capacitor has been charged to its full potential \( V_o \), it has an energy \( \frac{1}{2}CV_o^2 \) stored in it. Thus an energy \( \frac{1}{2}CV_o^2 \) has been dissipated while charging in the resistive parts of the circuit.
Figure 5.15:
Figure 5.16: R times the shaded area gives the energy dissipated

Figure 5.17: One step and two step charging voltage
Figure 5.18:

Figure 5.19:
Figure 5.20: