Study of a power supply

1.1 Introduction

A car battery can supply 12 volts. So can 8 dry cells in series. But no one would consider using the dry cells to start a car. Why not? Obviously, the dry cells cannot supply the large current required to start the car. The point is that the resistance of the source for the car battery (∼ 0.1 ohm) is considerably smaller than that for the 8 dry cells (∼ 5 to ∼ 70 ohms) in series*. A power supply which happens to be another commonly used source in the laboratory has a widely varying resistance; for a regulated power supply it may be as small as 0.1 ohm. A source of emf figure 1.1(a), therefore, must be represented not just by its voltage $V_s$ but by its source resistance $R_s$ as well figure 1.1(b). It is convenient to think of the source $V_s$ and its resistance $R_s$ as enclosed in an imaginary box (indicated by the dotted line in figure 1.1(b)) with terminals A and B, which we can put to any use we like. Electrical networks may be complicated but it is often very useful to think of parts of it as a 'box' with certain parameters associated with it—in the above case the parameters being $V_s$ and $R_s$.

* There are, of course, many other factors that dictate practical use of a power source. Consideration of cost, convenience of use, rechargeability, available power and energy etc. are some of these. For example, a dry cell may give only a few watt-hours of energy and cannot be recharged whereas a car battery can give 500 watt-hours and, with care, can be recharged any number of times. A power supply, on the other hand, derives its power continuously from the a.c. mains and hence needs no charging and can deliver any amount of energy. We shall however, not discuss these factors here, important as they are.
Figure 1.1:

Suppose we are given such a box with terminals A and B and we have to determine \( R_s \) and \( V_s \). First let us see how to do this in principle. We connect a voltmeter of very high resistance (ideally infinite) so that it draws no current. It will measure \( V_s \) directly. We can now connect an ammeter (ideally zero resistance) and measure the current which will be

\[
i = \frac{V_s}{R_s}
\]  

(1.1)

Thus, we may define the source resistance as the open circuit voltage between A and B divided by the current when A and B are short-circuited. In practice, we may have to exercise caution since the short circuit current may be very large and damage the instrument or the source itself.

We may now adopt the following attitude. The terminals A and B provide a certain source of voltage \( V_s \) with a source resistance \( R_s \). Actually, \( R_s \) may include other circuit elements as well. For example, think of the arrangements
Figure 1.2:
in figure 1.1(a) and figure 1.1(b). For these too we can represent the 'source' by a certain output voltage $V_s$ and source resistance $R_s$ as shown in figure 1.1(b). For the case of figure 1.1(a) Ohm's law gives us

$$V_s = V_o \frac{R_s}{R_1 + R_2}, R_s = \frac{R_1 R_2}{R_1 + R_2} \quad (1.2)$$

We can now say that we have a source of output voltage $V_s$ across the terminals AB, with an effective resistance $R_s$. This effective source resistance $R_s$ is often called the output resistance of the device as seen from AB. We shall develop the above ideas with a few simple experiments.
1.2 The Network Board-1

The network board for our experiment is shown in Plate 1. It contains three groups of resistors $R_1, R_2, R_3$, each group having several different resistors to choose from. It has a d-c milliammeter and a d-c voltmeter. Figure 1.3 shows the details of connections provided underneath the board. It will be seen that one could choose any one resistor from group $R_1$ and any one from group $R_2$ to make up a 'source' like that in figure 1.1(a). The third set of resistors $R_3$ are all connected in series and can be used as load. One could plug-in at any pair of points and get the desired value of the load.

1.3 Experiment A

To obtain the output voltage and output resistance of a given source.

![Diagram](image)

Figure 1.4:

Let the dotted 'box' in the figure 1.4 with AB for its output terminals be our 'source'. As can be seen from the figure, in fact it consists of a power supply of voltage $V_0$ and a potential divider arrangement made of resistors $R_1$ and $R_2$. We have to measure its output voltage across AB and then calculate its output resistance $R_s$. 
$V_s$ is measured by connecting the voltmeter directly across AB. Of course, it is implied here that the resistance of the voltmeter is so large that the current flowing through it can be neglected.

Now connect a resistor $R_L$ called load resistor along with a milliammeter. The current $i$ drawn from the source is measures by the milliammeter and the new voltmeter reading $V_L$ would be lower than $V_s$. If $R_s$ be the output resistance of the source then

$$V_s - iR_s = V_L \tag{1.3}$$

Thus the output resistance $R_s$ is given by

$$R_s = \frac{\text{Drop in output voltage}}{\text{Load current}} = \frac{V_s - V_L}{i} \tag{1.4}$$

If we take several different values of $R_L$, we shall be drawing different currents $i$. The voltage drop $V_s - V_L$ will also correspondingly change. You may tabulate these values, compute $R_s$ each time from eq(1.4), and obtain the mean $R_s$. Alternatively, you may draw a graph between $V_L$ and $i$ as shown in figure 1.5, see if it is a straight line, and obtain $R_s$ from its slope and $V_s$ from its intercept on the $V_L$ axis (since $i=0$ for this intercept $V_s$ would be the same as $V_L$). Can you appreciate why it is much better to calculate $R_s$ from the graph rather than directly from your observations?

Represent your results $V - s, R - s$ with a diagram like that in figure 1.1(b). This would be the 'equivalent circuit' for the actual source in fig 1.4.

### 1.4 Experiment B

To study the variation of the output resistance $R_s$ with changes in values of $R_1$ and $R_2$, the ratio $R_1/R_2$ remaining constant.

In the arrangement of figure 1.4, if the power supply is of voltage $V_o$ and resistance zero, then by ohm’s law the output voltage across AB should be

$$V_s = V_o \frac{R_2}{R_1 + R_2} \tag{1.5}$$

You may check the measured $V_s$ against the value calculated from eq(1.5). The dependence of the value of the output resistance $R_s$ on $R_1$ and $R_2$ is
obvious. Eq(1.5) shows that if both $R_1$ and $R_2$ are changed by the same factor, $V_s$ does not change. The value of $R_s$ should, however, change. Let us examine this by experiment.

With one set of $R_1$ and $R_2$ measure $R_s$ as in Expt A. Now change the resistors $xR_1$ and $xR_2$ where $x$ is some common factor. Measure $R_2$ again. Repeat this with different values of $x$ and examine how $R_s$ varies with $x$.

The behavior of $R_s$ with $x$ can be discussed as below. Figure 1.6(a) and figure 1.6(b) are equivalent. One can now see from the latter that if the power supply itself has a negligible resistance, then inside the 'source' $R_1$ and $R_2$ are in parallel, so that the output resistance of the source as seen at AB should be

$$R_s' = \frac{R_1 R_2}{R_1 + R_2}$$

(1.6)

By changing both $R_1$ and $R_2$ by a factor $x$, the new output resistance $R_s'$ will
be given by
\[ R'_s = \frac{xR_1 R_2}{xR_1 + xR_2} = \frac{xR_1 R_2}{R_1 + R_2} = xR_s \]  \hspace{1cm} (1.7)
Thus for a given ratio of \( R_1/R_2 \) i.e. for a given ratio of \( V_s/V_0 \), the output resistance comes out to be proportional to \( x \).

Using Ohm’s law, deduce an expression for the current \( i \) drawn by a load \( R_L \) connected across \( AB \) in figure 1.6(a). Use this result to obtain expressions for \( V_s \) and \( R_s \).

1.5 Experiment C

To study the power delivered by a source at different loads

A load resistor, connected across the terminals of a ‘source’, draw some current from it and thus consumes the power delivered to it by the ‘source’. It is interesting to study how the latter varies with the load. If, for a certain load \( R_L \), the current is \( i \), the voltage across the load is \( V_L \), then the power \( P \) delivered by the ‘source’ is given by

\[ P = V_L i \]  \hspace{1cm} (1.8)

Connect different resistors \( R_L \) and measure current \( i \) and voltage \( V_L \) each time figure 1.7. Tabulate these data and compute the power \( P \) using eq(1.8). Also plot against \( R_L \) and draw a smooth curve through the observation points figure 1.8.

The curve has a broad maximum for some value of \( R_L \). What is so special about this particular load? If you measure the output resistance \( R_s \) of the source(Expt. A) you will find that the power delivered is maximum when the load \( R_L \) has the same value as \( R_s \).

1.6 Experiment D

To learn more about ‘load matching’ and power dissipation in a circuit

We have already seen in Expt B how for a given ratio of \( R_1/R_2 \), the output voltage \( V_s \) does not depend on the individual values of \( R_1 \) and \( R_2 \).
whereas the output resistance $R_s$ does. Using this knowledge try different arrangements of $R_1$ and $R_2$. Measure the output resistance $R_s$ in each case (Expt A). Also measure the variation of the power $P$ delivered to the load $R_L$ for each value of $R_s$.

Plot the following quantities as a function of the load: (a) the load current $i$ (b) the voltage $V_L$ across the load (c) the power dissipated in the load (d) the fraction of power dissipated in the load upon power expended by the source. You will see that maximum power is delivered when the load $R_L$ is equal to the output resistance $R_s$. This disarmingly simple result is of great importance and you will come across it again and again in various forms. The idea is that the load resistance should match the output resistance for maximum power transfer*. You will also notice that the current $i$ is maximum when $R_L$ is zero, the voltage $V_L$ across the load is maximum when $R_L$ is infinite but the power $iV_L$ dissipated in the load is maximum when $R_L = R_s$ and is equal to $V_s^2/4R_s$. Remember, this is not the power expended by the source which is $V_s^2/2R_s$.

A source of output voltage $V_s$ and output resistance $R_s$ when connected across a load $R_L$ gives a current $i = \frac{V_s}{R_s + R_L}$ and delivers power to the load directly given by $P = i^2R_L$. Show, using differential calculus, that $P$ is maximum when $R_L = R_s$.

1.7 Experiment E

To study the reflected load resistance in a network**

Consider figure 1.9(a). When $R_L$ is not connected, let the current through the circuit be $i$. On connecting $R_L$, this current increases to some value $i_o$ which means that the load $R_L$ connected across AB increases the current from $i$ to $i_o$.

We can achieve the same result if we connect a suitable load $R'_L$ across

*This statement is true for alternating current circuits also. There we talk of output impedance instead of output resistance and the principle assumes its general name-the principle of impedance matching.

**For doing this experiment, you will need a resistance box in addition to the Network Board as shown in figure 1.3. Also note that in experiment on reflected load resistance measurement a power supply with an output voltage $V_o$ and negligible output resistance is used as the source.
CD (which means directly across the power supply). This load $R'_L$ seen by the source is called as the reflected load resistance.

For the simple circuit shown in figure 1.9 you can also calculate the reflected load resistance $R_L$ by applying Ohm’s law but in more complex networks such calculations may not be all that simple. Nevertheless, the fact remains that for a load $R_L$ across any two points (AB) in a network, simple or complex, you can always determine the reflected load $R'_L$ as seen by the power supply (across CD). In this sense therefore, the network acts as a ‘transformer’. It is usually called an impedance transformer when the network has components other than pure resistance also.

More generally, we can replace the actual load $R_L$ by a load $R'_L$ in another part of the circuit such that the current drawn from the source is the same. One then calls $R'_L$ as the transfer load (transfer resistance or transfer impedance as the case may be). An example of this is shown in figure 1.10.

Deduce an expression for the current in figure 1.9(a) when $R_L$ is connected. Deduce a similar expression for the case of figure 1.9(b) when $R'_L$ is connected. Hence, obtain an expression for the reflected load resistance. Draw conclusions for the limiting cases of $R_L \to \infty$ and $R'_L \to \infty$.

In the study of complicated circuits impedance transformations lead to considerable simplicity of analysis and are widely resorted to. We should, therefore, try to see this at least in a simple circuit like the one shown in figure 1.11.

Use a resistance box for $R_L$ along with the network board for this experiment. First keep $R'_L = \infty$ (plug off) and read the current in the milliammeter when $R_L = \infty$ and when $R_L$ has a given value. Let these readings be $i$ and $i_o$. Now set $R_L = \infty$, plug-in $R'_L$ and adjust its value such that the current has a value of $i_o$. Read the value of $R'_L$ at this stage.

Keeping $R_1, R_2$ and $R$ unchanged, take different values of load $R_L$ and for each case experimentally obtain the transfer load. It may be worthwhile to plot $R'_L$ against $R_L$ and see how it varies with $R_L$.

In the circuit of figure 1.9(a), the load $R_L$, on being connected across the output terminals AB, increases current from $i$ to $i_o$. Show that the transfer load (reflected resistance) $R'_L$ as seen at CD is given by $R'_L = \frac{V_s}{i_o-i}$ where $V_s$ is the output voltage at AB. (Thus $R'_L$ can be deduced from measurements $i, i_o$ unlike the method of direct substitution suggested in Expt E)
1.8 Experiment F

To make a simple equivalent circuit for a power 'source' ***

In Experiment A, you took a simple source figure 1.4 of output voltage $V_s$ and output resistance $R_s$. Now you may take a far more complicated arrangement, like the one shown in figure 1.12(a) and measure its $V - s$ and the series resistance is adjusted to be $R_S$. This is an 'equivalent circuit' corresponding to the circuit of figure 1.12(a). We may check this equivalence directly by experiment.

Make any network and choose any two points AB in that network as the 'output terminals'. Apply different loads $R_L$ at these terminals and each time measure the current i drawn and the voltage $V_L$ across AB. From these calculate $V_s$ and $R_s$ as in Expt A. Now take a power-supply and adjust its voltage to the value $V_s$. Connect a resistor of value $R_s$ in series with it Figure 1.12(b). Then apply the same loads $R_L$ across its output terminals AB and each time measure i and $V_L$. Compare these results with those obtained with the complicated network and see if the equivalence is complete.

Even when there is more than one source of emf in the network, the equivalence holds. In a-c circuits, with inductors and capacitors also present, the equivalence involves some more details, but is still a very useful concept.

*** This could be done immediately after Expt A as an exercise to see how any 'source'(with whatever complicated details) can be replaced by an equivalent circuit of an emf $V_s$ and a series resistance $R_s$. You would need some extra resistors in addition to your Network Board for doing this experiment.
APPENDIX

Carbon Resistors

Carbon, either alone or in combination with other materials, is used in making a class of resistors which are commonly used in radio and other communication circuits. After the advent of transistors and integrated circuits where one seldom handles large power, their use has gone up phenomenally. The commonest form of mass-produced resistors is the composition resistor, in which the conducting material, graphite or some other form of carbon, is mixed with fillers that serve as diluents and combined with an organic binder. Two general types of composition resistors are the solid body, which is moulded or extruded, and the filament type, in which carbon is baked on a glass or a ceramic rod and sealed in a ceramic or bakelite tube.

Composition resistors of the usual type are, however, notoriously unstable in resistance values. If they are used only at a low power level, the change in resistance results principally from the effect of humidity on the unit. If operated near the rated load, the changes in resistance result primarily from decomposition of the organic binder.

Much better stability is found in a special film type of resistor known as a pyrolitic or "cracked carbon" resistor. Such resistors are made by depositing crystalline carbon at a high temperature on a ceramic rod by "cracking" an appropriate hydrocarbon. In one process for making these film resistors, carbon is deposited from methane gas in a nitrogen atmosphere from which water vapour and oxygen are carefully excluded. No binder is used, and the carbon deposits consist of a hard gray crystalline form from which graphite and carbon black are completely absent. After the deposit is formed, the resistor is adjusted to its required value by cutting a helical groove around the cylinder with a diamond impregnated copper wheel. This removes part of the deposit and leaves a helical conductor of a suitable length and width for the desired resistance. After terminals are applied by a suitable process, the surface of a resistor is lacquered with some silicon type of varnish to provide insulation, moisture resistance and mechanical protection. These are then sorted out by measurement with a bridge in series having a tolerance of 10% or 5% or less.

For 1 watt 10k resistors of this type, a typical temperature coefficient is -0.02% per °C (minus sign indicates a decrease of resistance with increase in temperature unlike the wire-wound resistors) and for a 5megaohm resistor
this figure is -0.04% per °C.

There are some other advantages in using these film resistors. Their compactness of shape and size renders them easier to handle and suitable to fit in a small space. They are available over a wide range of resistance (from 1 ohm to 1000 megaohms or more). The 10% series are available in values starting from 1 ohm in a geometric progression of about 1.5 namely, 1, 1.5, 2.2, 3.3, 4.9... ... .... Similarly the values in the 5% series are in a geometric progression of 1.2 namely, 1, 1.2, 1.5, 1.8, 2.2... ... .... The resistance values are sometimes marked with colour bands. The colour code is-

Black 0, Brown 1, Red 2, Orange 3, Yellow 4, Green 5, Blue 6, Violet 7, Gray 8.

A simple way to remember this is the mnemonic - B.B. Roy Goes to Bombay Via Gateway [Perhaps you could make a better one].

You will notice four coloured bands, three narrow and one broad, on such resistors. The first two narrow ones, represent the first two numbers, and the third represents the number of zeroes after two numbers. The three narrow bands thus give the value of the resistance. The (fourth) broad one representing the tolerance is either a silver or a gold band, the former for 10% and the latter for 5%. Suppose on a resistor the colour bands are like this. Brown, Gray, Red and Gold. This would mean a value of 1800 i.e. 1.8k with 5% tolerance.

REFERENCE

Figure 1.6:
Figure 1.7:

Figure 1.8:
Figure 1.9:
Figure 1.10:

Figure 1.11:
Figure 1.12: