12.

Diffraction grating

OBJECT

To determine the wavelengths of light emitted by a mercury vapour lamp by using a diffraction grating.

INTRODUCTION:

Consider a light beam transmitted through an aperture in an opaque screen (see Fig. 12.1). If light were treated as rays traveling in straight lines, then the transmitted light would appear as a 'bright shadow' of the aperture. However, because of the wave nature of light, the transmitted pattern may deviate slightly or substantially from the aperture shadow. depending on the distance between the aperture and the observation plane, the dimensions of the aperture and the wavelength of light. Indeed, the transmitted intensity distribution, which is known as the *diffraction pattern*, may contain intensity maxima and minima even well outside the aperture shadow (see Fig. 12.1). The angles at which the intensity maxima and the minima occur depends on the wavelength of light and the width of the slit. This phenomenon of spreading out of light waves into the geometrical (dark) shadow when light passes through a small aperture (or about an obstacle) is known as *diffraction*.

A *diffraction grating* consists of a periodic array of a large number of equidistant slits of width 'b' which are separated by a distance 'a' as shown in Fig. 12.3. The period ($= a$) is known as the *grating constant*. Thus if N is the number of slits per unit length (say, 1 mm), then $a = I/N$ mm

The diffraction pattern due to a grating is essentially the same as the diffraction pattern due to M slits, where M is a large number ($\sim 10^3$) and is obtained by the superposition of waves emanating from all the slits on the observation plane. The resulting intensity distribution is given by

$$I = I_0 \left(\frac{sin\beta}{\beta}\right)^2 \left(\frac{\sin M\alpha}{sin\alpha}\right)^2 \qquad (12.1)$$

where

$$\alpha = \frac{ka\sin\theta}{2} \text{and } \beta = \frac{kb\sin\theta}{2} \qquad (12.2)$$

with $k = \frac{2\pi}{\lambda}$, λ being the wavelength of light and θ is the angle at which the diffracted beam propagates relative to the incident beam.

The grating equation

Consider the incidence of plane waves making an angle θ_i with the plane of the grating as shown in Fig. 12.3. The net path difference for waves from successive slits is given by

$$\Delta = \Delta_1 + \Delta_2 = a\sin\theta_i + a\sin\theta \qquad (12.3)$$

where θ is the angle corresponding to any arbitrary direction of the diffracted light.

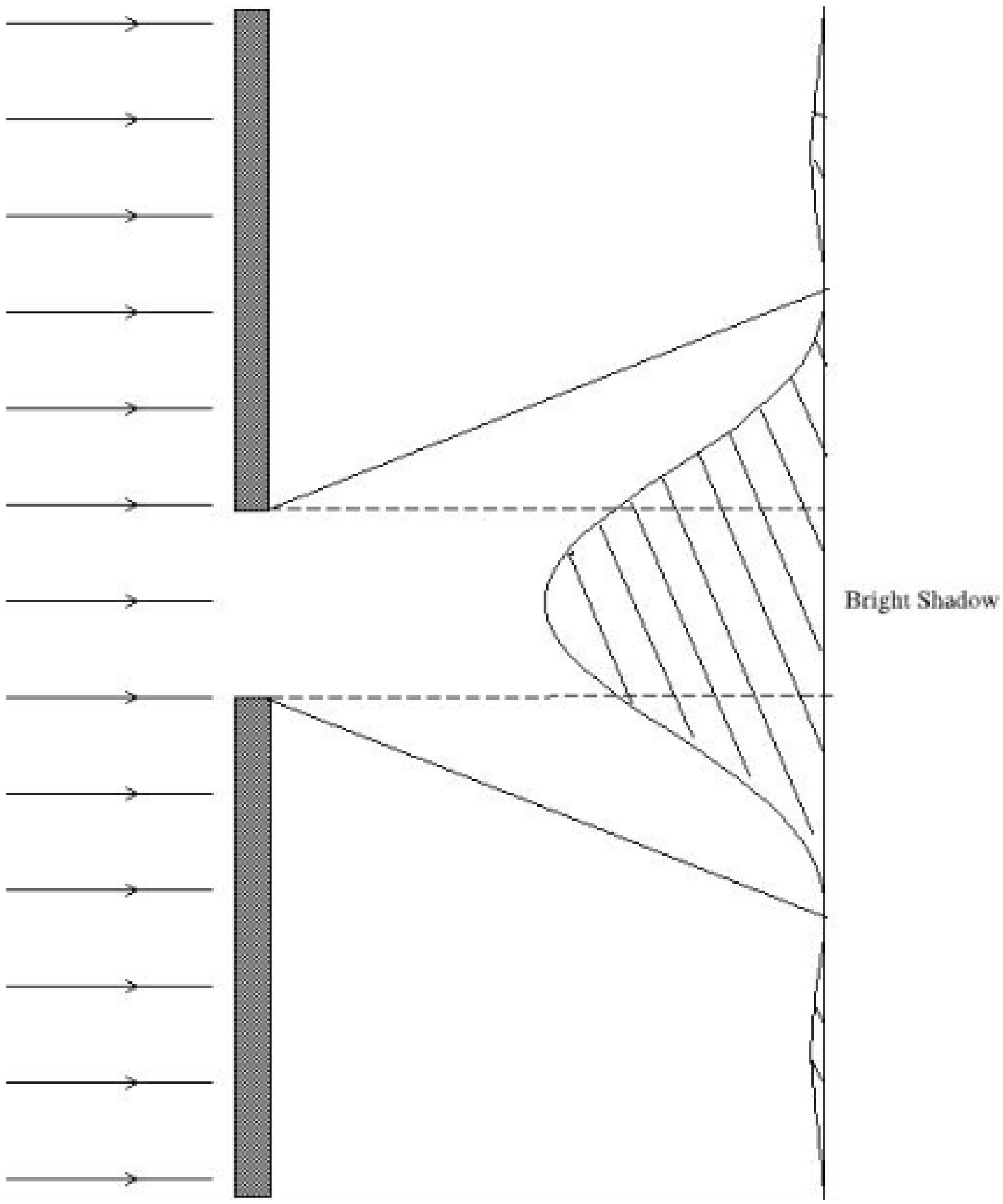

Figure 12.1: Diffraction of light through an aperture

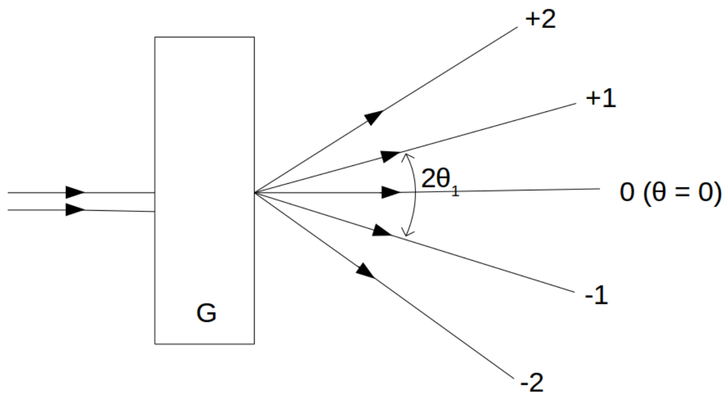

Figure 12.2: Diffracted orders at a particular wavelength

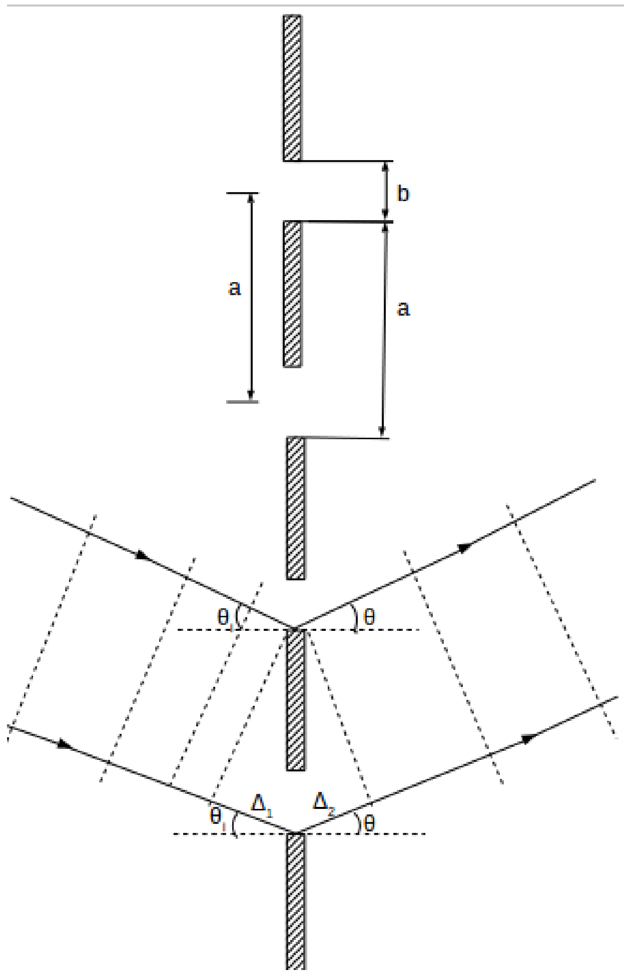

Figure 12.3: Plane waves incident on a diffraction grating at an angle θ_i 'a' is the grating constant

For normal incidence $\theta_i = 0$ and therefore

$$\Delta = a \sin\theta \tag{12.4}$$

When $\Delta = n\lambda$, where n is an integer, all diffracted waves in the corresponding direction θ_n are in phase, and their amplitudes add up to give maximum intensity. Thus, we have the grating equation, which gives the positions of the intensity maxima as

$$a \sin \theta_n = n\lambda \tag{12.5}$$

n = 0, ±1, ±2,... refers to the order of the spectrum.

The zeroth order (n = 0) occurs for $\theta_n = 0$, i.e. along the direction of the incident light, for all λ. Thus, light of all wavelengths appears in the zeroth order peak of the diffraction pattern. For orders $n \neq 0$, the grating leads to angular separation of the wavelengths present in the incident beam (see Eq. (12.5). In other words in each order, different colours would appear at different angles with reference to the direction of the incident beam. This feature of the granting makes it extremely useful in wavelength measurement and spectral analysis. Note that for every θ_n, satisfying the grating equation, the angle $-\theta_n$ also satisfies the grating equation with n replaced by -n. Thus, for normal incidence the +ve and -ve orders appear symmetrically on either side of the zeroth order (see Fig. 12.2).

Source of Light:

Mercury vapour lamp is used as the source of light. This source gives a well defined line spectrum arising from interstate electronic transitions taking place in the excited mercury atoms.

Spectrometer:

The spectrometer consists of a collimator which is mounted on the rigid arm, and a telescope mounted on the rotatable arm which can rotate in a horizontal plane about the axis of the instrument. A prism table of adjustable height is mounted along the axis of rotation of the telescope. A circular scale-and-vernier arrangement is provided to enable measurement of the angle through which the telescope arm or the prism table is rotated.

Experiment

1. Setting the prism table (This part is same as that for Expt. 8)

The prism table is made horizontal first with the help of a spirit-level by adjusting the leveling screws D, E and F (see Fig.4). To start with, the prism-table is rotated about its axis and adjusted in such a way that the parallel straight lines along with the two screws E and F are perpendicular to the axis joining the collimator and the telescope when they are aligned. A three way spirit level is kept on the prism table with its edge along the parallel lines.

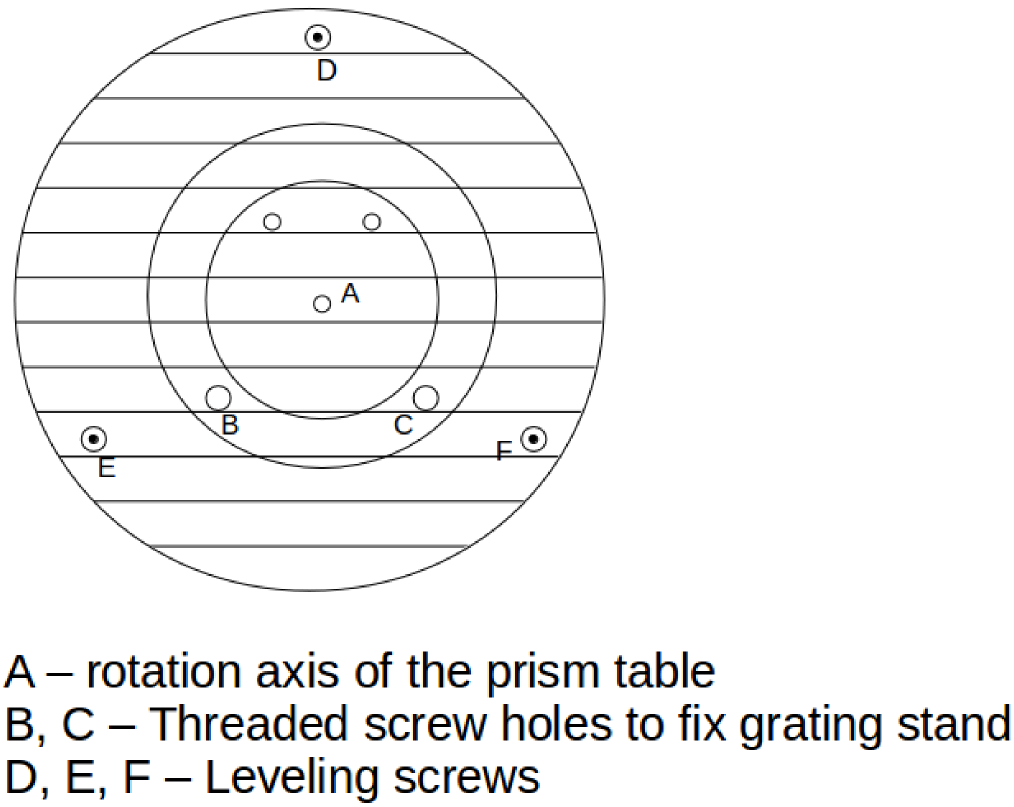

Figure 12.4: Top view of the prism table showing relevant details

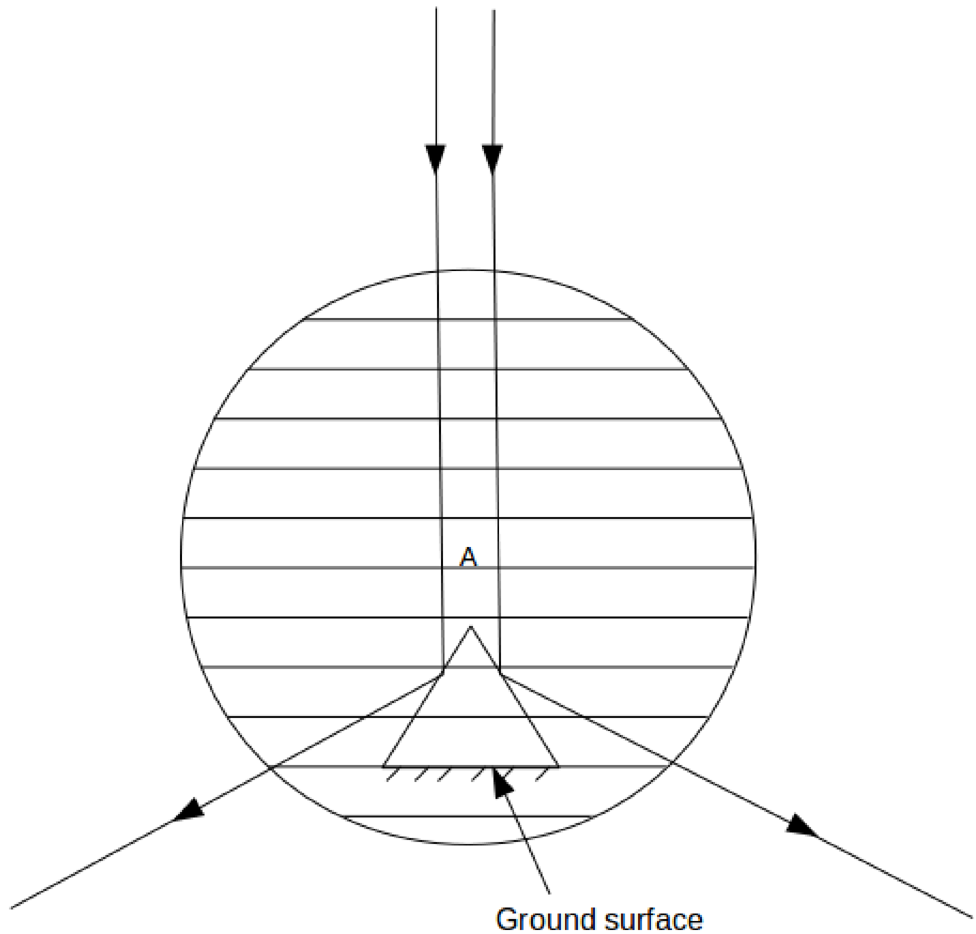

Figure 12.5: Positioning of the prism for optical alignment

Further adjustments of the prism table is done using the method of optical alignment. The given prism is placed such that the ground surface is facing towards the telescope and is perpendicular to the collimator. Adjust the position of the prism such that the edge of the prism opposite to the ground surface lies approximately along the axis of the prism table (see Fig.5). If you know rotate the telescope arm, you would be able to see the reflected images of the slit on both sides of the incident direction. Adjust the screws D, E and F such that the image of the slit appears symmetrically placed about the horizontal cross wire when viewed from both sides. The prism table adjustments are now complete.

2. Schuster's method of focusing a spectrometer for parallel incident light: (This part is same as for Expt. 8)

When a distant object is not available or if the spectrometer is too heavy to be carried outside the dark room where the experiment is being performed, the setting of the spectrometer is done by the so called Schuster's method.

First , the entrance slit of the collimator is kept facing the brightest porting of the mercury lamp and its width adjusted to permit a thin line of light to act as incident light. The given prism is now placed on the vernier table with its ground face along the parallel lines ruled on the prism table. The prism table is rotated so as to obtain mercury light incident from the collimator on one of the polished surfaces of the prism. The telescope arm is moved to a suitable position to see the spectrum through it (see Fig.6). The vernier table is rotated to achieve the position of minimum deviation.)of course, you will have to rotate the telescope arm also, as you rotate the vernier table, to retain the spectrum in the field of view of the telescope.) At this position, the spectrum which appeared to be moving (in the telescope) in one direction (say, left to right) reaches an extreme limit and retraces its path on further movement of the vernier table in the same direction.

Keeping the position of the telescope fixed, the vernier table is rotated slightly away from this position of minimum deviation, bringing the refracting angle towards the telescope and the telescope is now focused (see 1 - 1 in Fig.6) on the image as distinctly as possible. The vernier table is then rotated to the other side of the minimum deviation position towards the collimator and the collimator is focused (see 2 - 2 in Fig.6) to obtain a sharp image of the spectrum. The process is repeated till the motion of the prism does not effect the focus of the spectral lines.

The collimator and the telescope are then set for parallel light ant *these settings are not be disturbed during the course of the experiment.*

3. Setting up the diffraction grating for normal incidence:

The diffraction grating is positioned securely in the grating stand with the help of two clamps, and is fitted on the prism table with the help of two screws into the threaded holes B

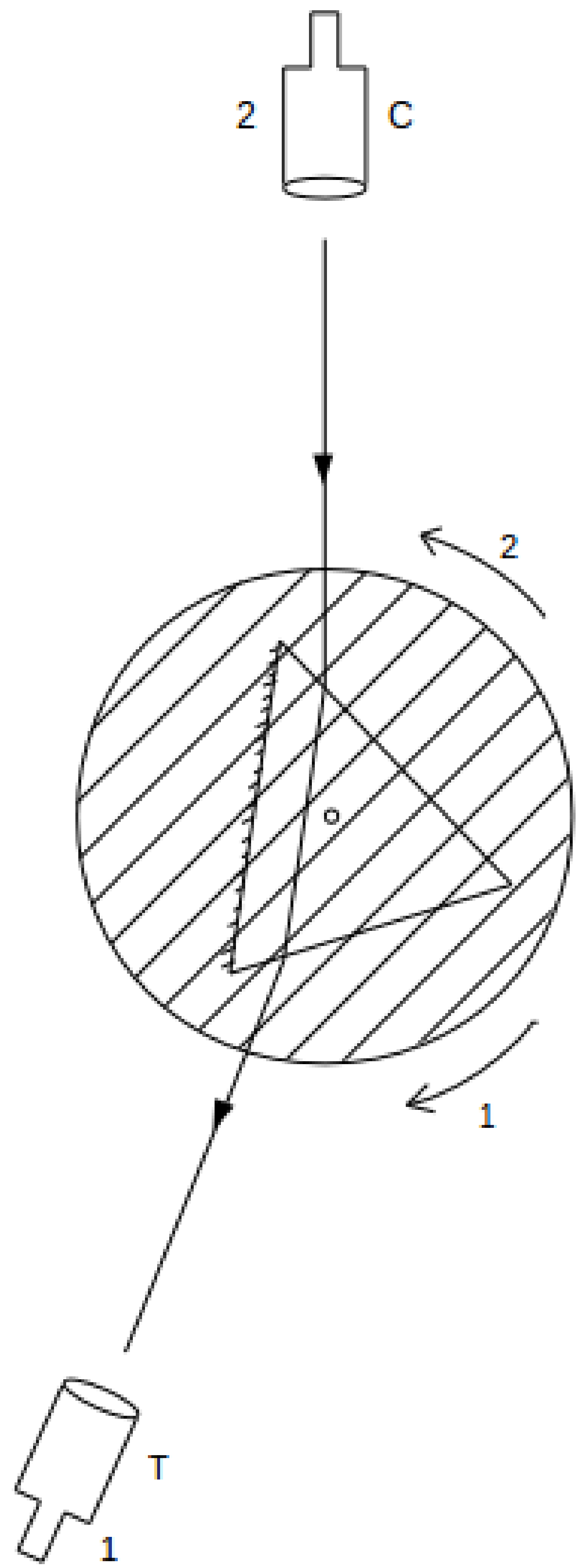

Figure 12.6: Top view of the setup for Schuster's method.

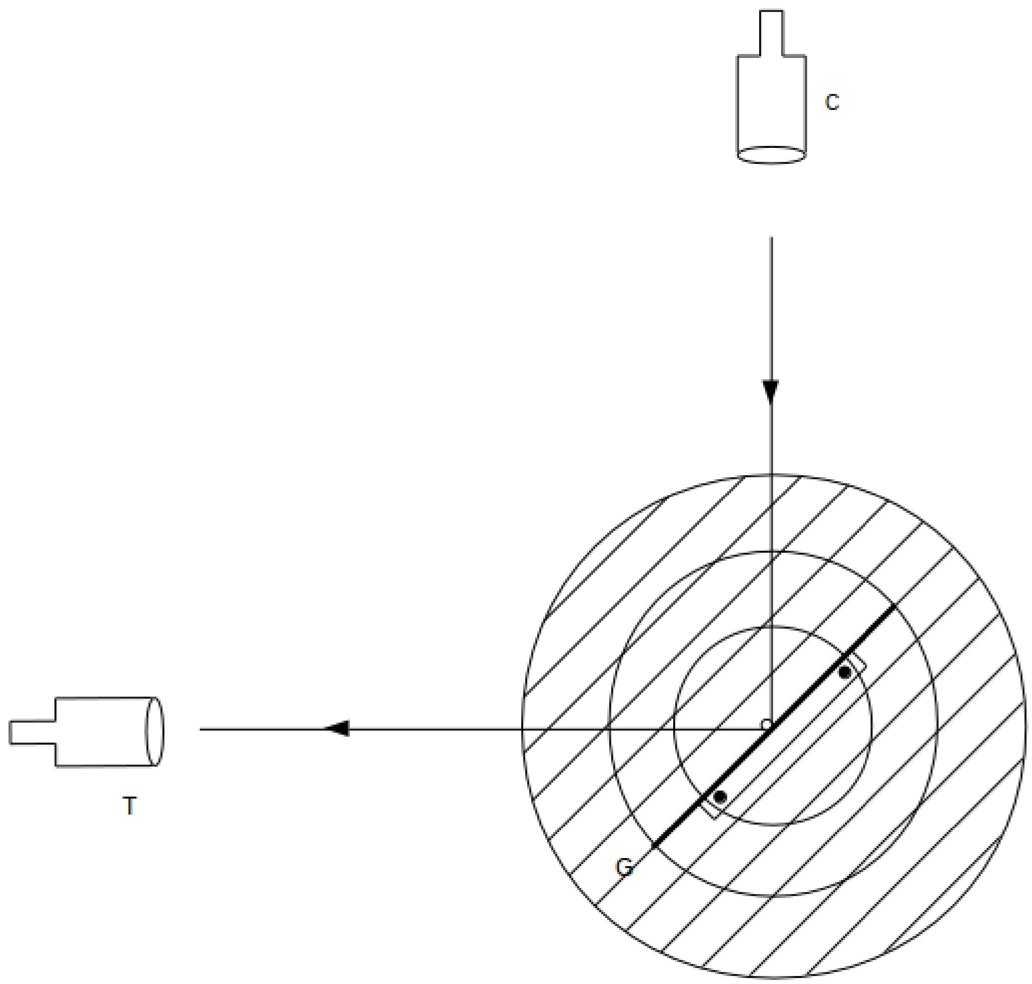

Figure 12.7:

and C, in Fig.4).

The position of the telescope is carefully adjusted such that the direct image of the slit coincides with the vertical crosswire on the telescope. Readings of the two circular scales I & II are recorded. The telescope arm is rotated through 90°, and locked in this new position. The prism table is rotated so as to coincide image of the slit reflected from the grating with the vertical crosswire in the telescope (see Fig.7). Readings of scales I and II are recorded again. The prism table is now rotated away from this position by an angle of 45° so as to make the grating face perpendicular to the incident light coming from the collimator. The prism table is locked in this position. The telescope arm is now released so that it can be moved freely on both sides of the incident light position.

4. Determination of Angle of Diffraction:

Experiment is performed with a grating having 2000 lines/inch or so. The diffraction spectrum contains a white line in the centre (zero order spectrum) with dispersed set of coloured lines (blue, blue green, green, yellow I, yellow II, red I, red II etc.) appearing repetitively on both sides of the zeroth order representing the higher orders of diffraction spectra. Readings of the telescope positions are taken while coinciding its crosswire with the various coloured lines on the left-side spectra. note down the readings of both the verniers for each spectral line in the first order and in the second order. Then take the telescope to the right side of the direct image and repeat the above procedure. Tabulate all the readings systematically as per the given format (see Table 1). Find out the differences in angles corresponding to the same kind of vernier for each spectral line in both the orders. Determine from this the wavelength of the light of a particular colour by using the grating formula

$$a sin\theta = n\lambda \tag{12.6}$$

Observations and calculations:

No. of rulings per inch on the grating 'N' = (given)
The grating constant $\alpha = ---$m = (Periodicity of the grating)
Least count of spectrometer =
Reading of telescope position for direct image of the slit =
Reading of telescope position after rotating it through 90° =
Reading on circular scale when the reflected image is obtained on the cross-wire =
Reading after rotating the prism tabel through 45° =

Order of Spectrum	Color of light	LHS reading for telescope position (p)	RHS reading for telescope position (q)	p − q (deg)	$\theta = \frac{\lvert p-q \rvert}{2}$ (deg)
2	Yellow Green Violet	↓	↑		
1	Yellow Green Violet	↓ →	↑		

*The direction of the arrow indicates the sequence of recording the readings.

Precautions:

i Care should be taken to ensure proper setting of the spectrometer and these settings of the telescope and the collimator are not touched during the course of taking the various readings.

ii The position of the grating adjusted to be normal to the incoming light from the collimator, should not be distrubed throughout the experiment. Ensure that the prism table locking screw is tightened properly.

iii It is necessary to point the slit towards the brightest part of the source, in order to obtain reasonable intensity of the lines of different colours especially in the higher order spectra. It is known that the intensity of lines in the higher order spectra reduces sharply with increase tn order.

Sources of error:

Think and find out yourself after doing the experiment!!