# 14.

# To determine the viscosity of water by Meyer's oscillating disk method

## Experiment 14(V)

### Theory

If a disk undergoes torsional oscillations about its symmetry axis in a fluid medium, it does not push aside any additional fluid while executing this motion. The fluid in contact with the disk then remains at rest with respect to it, while the fluid far away is at rest with respect to the enclosure/container. so a transverse velocity gradient is set up in the fluid, and this in turn causes a viscous force to act and damp out the oscillations. Oscar Meyer suggested measuring the decay of these oscillations to find the viscosity of a liquid.
The equation to a harmonic oscillator undergoing torsional os-

cillations is.

$$I\frac{d^2\theta}{dt^2} + K\frac{d\theta}{dt} + \tau\theta = 0. \tag{14.1}$$

Here I is the moment of inertia of the oscillator, K is the damping coefficient, $\tau$ is the restoring torque per unit twist and $\theta$ is the oscillations (twist) angle. The solution of this equation is given by,

$$\theta(t) = \theta_0 e^{-\frac{2\lambda t}{T}} sin(\frac{2\pi t}{T} + \phi), \lambda = \frac{KT}{4}, T = 2\pi I\left(\frac{1}{\tau - \frac{K^2}{4}}\right)^{\frac{1}{2}} \tag{14.2}$$

where $\theta_0$ and $\phi$ are constants of integration. The variation of this function with time is shown in the Fig 14.1. The quantity $\lambda$, known as the logarithmic decrement, is the logarithm of the ratio of any two successive amplitudes on opposite sides of the equilibrium position. Thus,

$$e^{\lambda} = \frac{B_1C_1}{B_2C_2} = \frac{B_2C_2}{B_3C_3} = \frac{B_1C_1 + B_2C_2}{B_2C_2 + B_3C_3} = \frac{B_1C_1 + B_2C_2... + B_nC_n}{B_2C_2 + B_3C_3... + B_{n+1}C_{n+1}} \tag{14.3}$$

Here $B_j$ is the amplitude at the $i^{th}$ turning point of the disk, as shown in Fig.1. Thus by measuring the amplitudes on either side of the equilibrium position, we can find out the damping coefficient using Eq.(14.3).

In the case of a disk oscillating inside a liquid, the damping is due to two causes: damping due to the viscous forces of the liquid, and damping due to the friction of the wire suspension at the support. Meyer suggested that the instrument be first used to find the logarithmic decrement $\lambda_0$ in air, where the viscous damping is negligible, followed by a measurement of the logarithmic decrement $\lambda$ in the liquid. As the frictional damping at the support is the same in both cases., this (unknown quantity) can be eliminated by taking the difference $\lambda - \lambda_0$. Using this, he was able to find a formula for the viscosity of the liquid as,

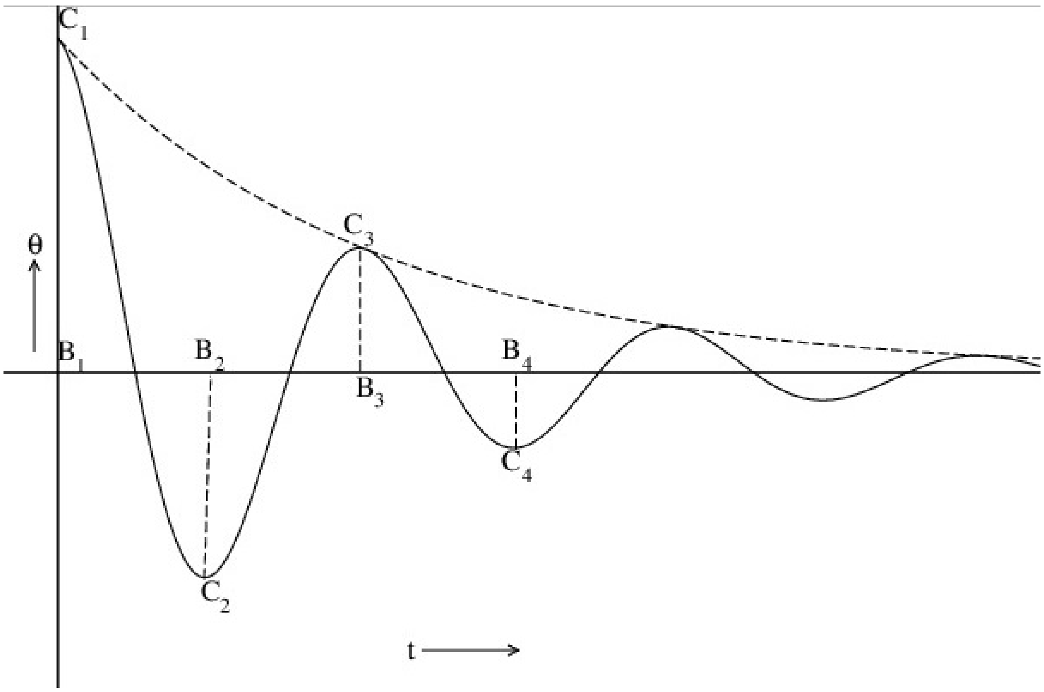


Figure 14.1: Damped Oscillations

$$\eta = \frac{16I^2}{\pi\rho T(r^4 + 2r^3 d)^2}\left[\left[\frac{\lambda-\lambda_0}{\pi}\right] + \left[\frac{\lambda-\lambda_0}{\pi}\right]^2\right]^2 \qquad (14.4)$$

Here,
I - moment of inertia of the torsional pendulum about the suspension axis.
T - time period for one complete oscillation.
r - radius of the disk.
d - thickness of the disk.

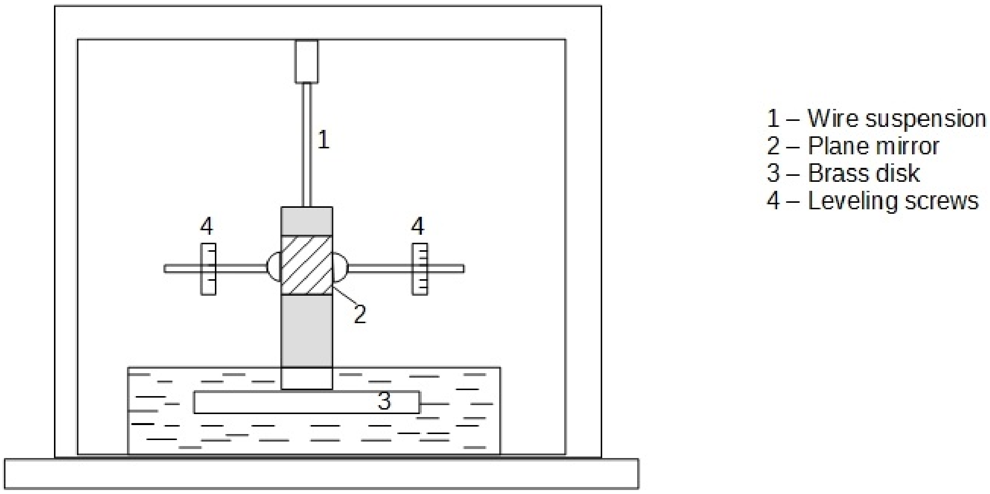


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Figure 14.2: Meyer's Apparatus

$\rho$ - density of the liquid.
$\lambda$ - logarithmic decrement in the liquid.
$\rho_0$ - logarithmic decrement in air.

The quantities mentioned above can all be measured directly, except the moment of intertia of the disk which is a complex object. To find the moment of inertia, the time period (T) of the disk in air is found and then a ring with a known moment of inertia $I_r$ is placed on the disk with its center on the suspension axis. The time period of the disk and the ring together in air(T') is again found, when the moment of inertia of the ring-loaded disk is $I + I_t$. Then, we have

$$I_r = ma^2 \qquad \therefore \quad I = ma^2 \frac{T^2}{(T')^2 - T^2} \tag{14.5}$$

$$T = 2\pi \left( \frac{I}{\tau - \frac{\kappa^2}{4}} \right)^{\frac{1}{2}} = \left( \frac{I}{\tau} \right)^{\frac{1}{2}} \qquad and \qquad T' = \left( \frac{I + I_r}{\tau} \right)^{\frac{1}{2}} \tag{14.6}$$

here, m is the mass, a is the average radius of the ring, i.e., $a = (d_1 + d_2)/4$ where $d_1$ and $d_2$ are the inner and outer diameters of the ring, respectively. Using equations (4) and.(6) we can find the viscosity of water.

**PROCEDURE**

**1.** The apparatus consists of a flat disk attached to a short rod passing through its center which is suspended (with the disk remaining horizontal) by means of a phosphor bronze wire. The central rod has a perpendicular screw with two movable masses on opposite sides for leveling the disk. A small concave mirror with a radius of curvature of about one meter is also mounted on this rod (see Fig.2).

A lamp and scale arrangement is provided which is to be adjusted till a beam from the lamp after reflection from the concave mirror forms a well defined circular patch of light on the scale. The image of the cross wires on the lamp should be clearly visible on the screen. The positions of the scale and the disk are adjusted till the equilibrium position of the spot of light is close to the center of the scale.

**2.** Taking care to avoid all transverse oscillations (such as lateral swing or wobble), the disk is rotated slightly to give a small torque and left free to undergo torsional oscillations. By measuring the time of 25 oscillations, the time period of the pendulum

T is found. Repeat this step once more and take the mean value of T.

**3.** The given metallic ring is placed flat on the disk, so that it's center is as close as possible to the axis of suspension. The time period of the pendulum T' is now found by the procedure described above. The mass of the ring, and the outer and inner diameters ($d_1$ and $d_2$) of the ring are measured. Make observation tables for these measurements. (The ring may not be exactly circular: therefore measure the diameter along different directions and take the average value). Using these two measurements and Eq.(6) the moment of inertia I of the pendulum can be calculated. The ring can now be removed and is not required in the rest of the experiment.

**4.** To measure the logarithmic decrement, the disk is again set into torsional oscillation. When the amplitude has fallen to approximately the full scale reading, start the readings by noting down the reading on the scale at one extreme position, $B_1C_1$. The very next reading at the outer turning point $B_2C_2$ is then recorded (see Fig.1).

**5.** After 20 complete oscillations, again record the maximum amplitudes on both sides $B_{41}C_{41}$ & $B_{42}C_{42}$. The logarithmic decrement in air can now be found by using these readings and Eq.(3) for 20 oscillations (i.e., for n=20) as,

$$\lambda_0 = \frac{1}{40} \ln \left( \frac{B_1C_1 + B_2C_2}{B_{41}C_{41} + B_{42}C_{42}} \right) \tag{14.7}$$

In general, if n is the number of oscillations, then the logarithmic decrement is the given by

$$\lambda_0 = \frac{1}{2n} \ln \left( \frac{B_1 C_1 + B_2 C_2}{B_{2n+1} C_{2n+1} + B_{2n+2} C_{2n+2}} \right) \qquad (14.8)$$

Repeat the procedure for 30 and 40 oscillations to calculate $\lambda_0$. Take the mean value of $\lambda_0$ to obtain the logarithmic decrement.

**6.** A clean glass dish is now placed so as to contain the disk, and water is poured into it so as to cover the disk but not submerge the mirror (see Fig.2). The equilibrium position of the light spot is now adjusted (if necessary) so that it again lies at the center of the scale. The same procedure (as that to find the logarithmic decrement in air) is now repeated to find the logarithmic decrement $\lambda$ in water. Since the oscillations in this case are very much damped, the experiment has to be performed for smaller number of oscillations.

**Tabulate the observation for air and water as shown in Table 1 and Table 2.**

**7.** Using the data measured above, and the dimensions of the disk equation (4) is used to find the viscosity of water. The temperature of the water used must be measured and quoted along with the result.

**Observations:**

Least count of vernier caliper used =
Least count of stop watch =
Least count of balance used =

Radius of the disk, r =
Thickness of the disk, d =
Outer diameter of the ring, $d_1$ =
Inner diameter of the ring, $d_2$ =
Average radius of the ring, a =
Mass of the ring, m =
Temperature of water =
Time required for 25 oscillations in air =
Time period in air, T =
Time required for 25 oscillations in air with ring =
Time period in air with ring, T' =

Table 14.1: Readings for finding logarithmic decrement in Air

| Trial number | Serial no. of oscillation | | Maximum Amplitude Left($B_iC_i$) | Right($B_{i+}$)$C_{i+1}$) | $\lambda_0$ {using Eq.(8)} |
|---|---|---|---|---|---|
| 1 | Start | (i= 1) | | | |
| | n = 20 | (i = 41) | | | |
| 2 | Start | (i= 1) | | | |
| | n = 30 | (i = 61) | | | |
| 3 | Start | (i= 1) | | | |
| | n = 40 | (i = 31) | | | |

Calculate the maximum probable error $d\eta$ and write down the precautions and sources of error.

**Result:**

The viscosity of water was found to be ________ poise, at a temperature of ________degrees centigrade.

Table 14.2: Readings for logarithmic decrement in Water

| Trial number | Serial No. of oscillation | | Maximum Amplitude Left ($B_iC_i$) | Right ($B_{i+1}C_{i+1}$) | $\lambda$ {using Eq.(8)} |
|---|---|---|---|---|---|
| 1 | Start | ( i= 1) | | | |
| | n = 5 | (i = 11) | | | |
| 2 | Start | (i= 1) | | | |
| | n = 10 | (i = 21) | | | |
| 3 | Start | (i= 1) | | | |
| | n = 15 | (i = 31) | | | |